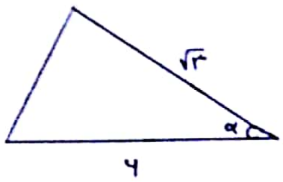
نام و نام خانوادگی ...آناهیتا ایمانی... پاسخنامه تشریحی تکلیف شماره ...27... کلاس ...یازدهم A...

۲۰

1



$S = \frac{1}{2}ab\sin\alpha$

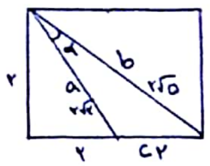
$\cancel{4/5} = \frac{1}{2} \times \sqrt{3} \times \cancel{6} \times \sin\alpha \rightarrow \sin\alpha = \frac{3}{2\times\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{6}$

$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2} \rightarrow \alpha = 60^\circ$ یا $\alpha = 120^\circ$

$\frac{120}{60} = \boxed{2}$ ۲ برابر

۲

2



$a = \sqrt{2^2+2^2} = 2\sqrt{2} \qquad b = \sqrt{2^2+4^2} = 2\sqrt{5}$

قضیه کسینوس ها $\Rightarrow c = \sqrt{a^2+b^2-2ab\cos\alpha} \rightarrow 2 = \sqrt{(2\sqrt{2})^2+(2\sqrt{5})^2-2(2\sqrt{2})(2\sqrt{5})\cos\alpha}$

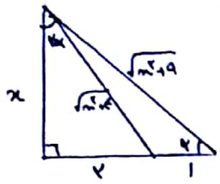
$\rightarrow 4 = 8 + 20 - 8\sqrt{10}\cos\alpha \rightarrow 8\sqrt{10}\cos\alpha = 24 \rightarrow \cos\alpha = \frac{24}{8\sqrt{10}} = \frac{3}{\sqrt{10}}$

$\cos^2\alpha + \sin^2\alpha = 1 \rightarrow \frac{9}{10} + \sin^2\alpha = 1 \Rightarrow \sin^2\alpha = \frac{1}{10} \Rightarrow \sin\alpha = \pm\frac{1}{\sqrt{10}} \qquad \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}} = \boxed{3}$

چون α زاویه ای در مثلث و کمتر از ۱۸۰ است، sin مثبت است $\Leftarrow \sin\alpha = \frac{1}{\sqrt{10}}$

۲

3



$\sin 2\alpha = \frac{2}{\sqrt{x^2+4}} \qquad \cos 2\alpha = \frac{x}{\sqrt{x^2+4}} \rightarrow \cot 2\alpha = \frac{x}{2}$

$\sin\alpha = \frac{x}{\sqrt{x^2+9}} \qquad \cos\alpha = \frac{3}{\sqrt{x^2+9}} \rightarrow \tan\alpha = \frac{x}{3}$

$\underbrace{\frac{\cos 2\alpha}{\sin 2\alpha}}_{\cot 2\alpha} = \frac{\frac{1-\tan^2\alpha}{\cancel{1+\tan^2\alpha}}}{\frac{2\tan\alpha}{\cancel{1+\tan^2\alpha}}} \Rightarrow \cot 2\alpha = \frac{1-\tan^2\alpha}{2\tan\alpha} \rightarrow \frac{x}{2} = \frac{1-\frac{x^2}{9}}{\frac{2x}{3}} = \frac{9-x^2}{6x} \rightarrow \frac{9-x^2}{9}\times\frac{3}{2x}$

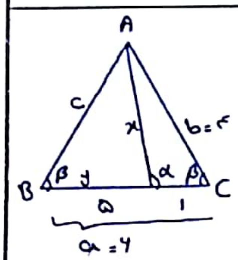
$3x^2 = 9 - x^2 \Rightarrow 4x^2 = 9 \rightarrow x^2 = \frac{9}{4} \Rightarrow x = \pm\frac{3}{2} \Rightarrow x = \frac{3}{2}$

غ ق ق

$\cot\alpha = \frac{1}{\tan\alpha} = \frac{3}{\frac{3}{2}} = 2$

۲

4



$c^2 = a^2 + b^2 - 2ab\cos\beta \rightarrow 16 = 36 + 16 - 48\cos\beta \rightarrow 48\cos\beta = 36 \rightarrow \cos\beta = \frac{3}{4}$

$x^2 = c^2 + y^2 - 2cy\cos\beta \rightarrow x^2 = 16 + 25 - (40)(\frac{3}{4}) \rightarrow x^2 = 11 \Rightarrow x = \sqrt{11}$

$b^2 = x^2 + 1 - 2x\cos\alpha \rightarrow 16 = 11 + 1 - 2\sqrt{11}\cos\alpha \rightarrow 4 = -2\sqrt{11}\cos\alpha \Rightarrow \cos\alpha = \frac{-2}{\sqrt{11}}$

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \sin^2\alpha + \frac{4}{11} = 1 \rightarrow \sin^2\alpha = \frac{7}{11} \Rightarrow \sin\alpha = \pm\sqrt{\frac{7}{11}} \rightarrow$ چون α زاویه ای در مثلث و کمتر از ۱۸۰ است، sin باید مثبت باشد $\Leftarrow \sin\alpha = \sqrt{\frac{7}{11}}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{\sqrt{7}}{\sqrt{11}}}{\frac{-2}{\sqrt{11}}} = -\frac{\sqrt{7}}{2}$

۲

5

$$\begin{cases} 2\sin^2x + \cos^2x = \frac{4}{3} \\ -\ \sin^2x + \cos^2x = 1 \end{cases}$$

$\sin^2x = \frac{1}{3} \rightarrow \sin^2x + \cos^2x = 1 \Rightarrow \cos^2x = \frac{2}{3}$

$\tan^2x = \frac{\frac{1}{3}}{\frac{2}{3}} = \boxed{\frac{1}{2}}$

۲

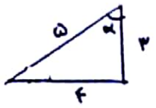
$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{(1-\cos^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{(1-\sin^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}$$

$$\frac{1-2\cos^2\alpha+\cos^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{1-2\sin^2\alpha+\sin^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{1+2\cos^2\alpha+\cos^4\alpha}{1+\cos^2\alpha}-\frac{1+2\sin^2\alpha+\sin^4\alpha}{1+\sin^2\alpha}$$ ۶

2

$$\rightarrow \frac{(1+\cos^2\alpha)^{\cancel{2}}}{\cancel{1+\cos^2\alpha}}-\frac{(1+\sin^2\alpha)^{\cancel{2}}}{\cancel{1+\sin^2\alpha}}=1+\cos^2\alpha-1-\sin^2\alpha=\cos^2\alpha-\sin^2\alpha=\boxed{\cos 2\alpha}$$

$\tan\alpha=\frac{4}{3}$



$\rightarrow$ چون انتهای کمان در ربع سوم است، Cos و Sin باید منفی باشند $\Rightarrow \sin\alpha=-\frac{4}{5}\quad \cos=-\frac{3}{5}$

$\cot\alpha=\frac{3}{4}$

$$\underbrace{\sin\left(\frac{9\pi}{2}+\alpha\right)}_{\cos\alpha}\underbrace{\cos\left(\frac{7\pi}{2}-\alpha\right)}_{-\sin\alpha}-\underbrace{\tan\left(\alpha-\frac{3\pi}{2}\right)}_{-\cot\alpha}=\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)+\frac{3}{4}=\frac{-12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\boxed{\frac{27}{100}}$$ ۷

2

$$\left(3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x\right)\xrightarrow{x=\frac{\pi}{12}}3\cos\left(\frac{\pi}{3}\right)+\sqrt{2}\sin\left(\frac{\pi}{12}\right)-\sqrt{2}\cos\left(\frac{\pi}{12}\right)\rightarrow$$

$$3\cos\frac{\pi}{3}+2\Big(\underbrace{\frac{\sqrt{2}}{2}}_{\sin\frac{\pi}{4}}\sin\frac{\pi}{12}-\underbrace{\frac{\sqrt{2}}{2}}_{\cos\frac{\pi}{4}}\cos\frac{\pi}{12}\Big)=3\cos\frac{\pi}{3}-2\Big(\underbrace{\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}}_{\cos\left(\frac{\pi}{4}+\frac{\pi}{12}\right)=\cos\frac{\pi}{3}}\Big)=3\cos\frac{\pi}{3}-2\cos\frac{\pi}{3}$$ ۸

$$=\cos\frac{\pi}{3}=\frac{1}{2}$$

2

$$\tan\left(\frac{\alpha}{2}\right)=\frac{1}{4}\rightarrow \tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}\rightarrow \tan\alpha=\frac{2\times\frac{1}{4}}{1-\frac{1}{16}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}$$

$$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\rightarrow 1+\frac{64}{225}=\frac{1}{\cos^2\alpha}\rightarrow\frac{289}{225}=\frac{1}{\cos^2\alpha}\rightarrow\cos\alpha=\frac{15}{17}$$

2

$$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}\rightarrow\frac{8}{15}=\frac{\sin\alpha}{\frac{15}{17}}\rightarrow\sin\alpha=\frac{8}{17}$$ ۹

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{8(17-15)}{15\times\cancel{17}}}{\frac{-7}{\cancel{17}}}=\frac{\frac{8\times2}{15}}{-7}=\boxed{-\frac{16}{105}}$$

$$\frac{\cot\alpha}{\sin\alpha}>0\rightarrow\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}>0\rightarrow\frac{\cos\alpha}{\sin^2\alpha}>0\rightarrow$$ چون $\sin^2\alpha$ همواره مثبت است پس $\cos\alpha>0$ (I)

2

$$2\sin\alpha<\sin2\alpha\rightarrow\cancel{2}\sin\alpha<\cancel{2}\sin\alpha\cos\alpha\rightarrow\sin\alpha<\sin\alpha\cos\alpha\Rightarrow\sin\alpha-\sin\alpha\cos\alpha<0\rightarrow\sin\alpha(1-\cos\alpha)<0$$

$\leftarrow$ چون $-1\le\cos\alpha\le1$، پس $(1-\cos\alpha)$ همواره مثبت است بنابراین $\sin\alpha<0$ (II) ۱۰

(I), (II) $\rightarrow \cos\alpha>0\ ,\ \sin\alpha<0\Rightarrow$ ربع چهارم