نام و نام خانوادگی ...النا هیئت ایمانی... پاسخنامه تشریحی تکلیف شماره ...۲۷... کلاس ...یازدهم تجربی A...

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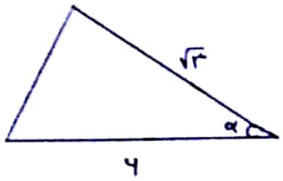

$S=\frac{1}{2}ab\sin\alpha$

$\cancel{4/5}=\frac{1}{\cancel{2}}\times\sqrt{3}\times\cancel{6}\times\sin\alpha \rightarrow \sin\alpha=\frac{3}{2\times\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{6}$

$\Rightarrow \sin\alpha=\frac{\sqrt{3}}{2} \rightarrow \alpha=60^\circ$ یا $\alpha=120^\circ$

$\frac{120}{60}=\boxed{2}$ ۲ برابر

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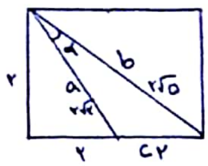

$a=\sqrt{2^2+2^2}=2\sqrt{2}$ $\quad b=\sqrt{2^2+4^2}=2\sqrt{5}$

قضیه کسینوس ها $\Rightarrow c=\sqrt{a^2+b^2-2ab\cos\alpha} \rightarrow 2=\sqrt{(2\sqrt{2})^2+(2\sqrt{5})^2-2(2\sqrt{2})(2\sqrt{5})\cos\alpha}$

$\rightarrow 4=8+20-8\sqrt{10}\cos\alpha \rightarrow 8\sqrt{10}\cos\alpha=24 \rightarrow \cos\alpha=\frac{24}{8\sqrt{10}}=\frac{3}{\sqrt{10}}$

$\cos^2\alpha+\sin^2\alpha=1 \rightarrow \frac{9}{10}+\sin^2\alpha=1 \Rightarrow \sin^2\alpha=\frac{1}{10} \Rightarrow \sin\alpha=\pm\frac{1}{\sqrt{10}}$ $\quad \cot\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}}=\boxed{3}$

$\sin\alpha=\frac{1}{\sqrt{10}}$ ⇐ چون α زاویه ای کمتر از ۱۸۰ است، sin مثبت است

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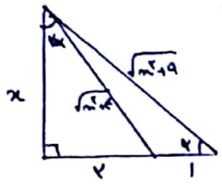

$\sin2\alpha=\frac{2}{\sqrt{x^2+4}}$ $\quad \cos2\alpha=\frac{x}{\sqrt{x^2+4}} \rightarrow \cot2\alpha=\frac{x}{2}$

$\sin\alpha=\frac{x}{\sqrt{x^2+9}}$ $\quad \cos\alpha=\frac{3}{\sqrt{x^2+9}} \rightarrow \tan\alpha=\frac{x}{3}$

$\cot2\alpha=\frac{\cos2\alpha}{\sin2\alpha}=\frac{\frac{1-\tan^2\alpha}{\cancel{1+\tan^2\alpha}}}{\frac{2\tan\alpha}{\cancel{1+\tan^2\alpha}}} \Rightarrow \cot2\alpha=\frac{1-\tan^2\alpha}{2\tan\alpha} \rightarrow \frac{x}{2}=\frac{1-\frac{x^2}{9}}{\frac{2x}{3}}=\frac{9-x^2}{6x} \rightarrow$

$3x^2=9-x^2 \Rightarrow 4x^2=9 \rightarrow x^2=\frac{9}{4} \Rightarrow x=\pm\frac{3}{2} \Rightarrow x=\frac{3}{2}$

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$\cot\alpha=\frac{1}{\tan\alpha}=\frac{3}{\frac{3}{2}}=2$

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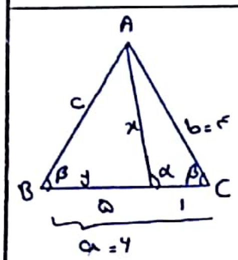

$c^2=a^2+b^2-2ab\cos\beta \rightarrow 16=36+16-48\cos\beta \rightarrow 48\cos\beta=36 \rightarrow \cos\beta=\frac{3}{4}$

$x^2=c^2+y^2-2cy\cos\beta \rightarrow x^2=16+25-(40)(\frac{3}{4}) \rightarrow x^2=11 \Rightarrow x=\sqrt{11}$

$b^2=x^2+1-2x\cos\alpha \rightarrow 16=11+1-2\sqrt{11}\cos\alpha \rightarrow 4=-2\sqrt{11}\cos\alpha \Rightarrow \cos\alpha=\frac{-2}{\sqrt{11}}$

$\sin^2\alpha+\cos^2\alpha=1 \rightarrow \sin^2\alpha+\frac{4}{11}=1 \rightarrow \sin^2\alpha=\frac{7}{11} \Rightarrow \sin\alpha=\pm\sqrt{\frac{7}{11}} \rightarrow$ چون α زاویه ای کمتر از ۱۸۰ است، sin باید مثبت باشد $\Rightarrow \sin\alpha=\sqrt{\frac{7}{11}}$

$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{\sqrt{7}}{\sqrt{11}}}{\frac{-2}{\sqrt{11}}}=-\frac{\sqrt{7}}{2}$

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$\begin{cases}2\sin^2x+\cos^2x=\frac{4}{3}\\ -\quad \sin^2x+\cos^2x=1\end{cases}$

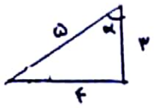
$\sin^2x=\frac{1}{3} \rightarrow \cancel{\sin^2x}+\cos^2x=1 \Rightarrow \cos^2x=\frac{2}{3}$

$\tan^2x=\frac{\frac{1}{3}}{\frac{2}{3}}=\boxed{\frac{1}{2}}$

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{(1-\cos^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{(1-\sin^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}$$

$$\frac{1-2\cos^2\alpha+\cos^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{1-2\sin^2\alpha+\sin^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{1+2\cos^2\alpha+\cos^4\alpha}{1+\cos^2\alpha}-\frac{1+2\sin^2\alpha+\sin^4\alpha}{1+\sin^2\alpha}$$ ۶

$$\rightarrow \frac{(1+\cos^2\alpha)^{\cancel{2}}}{\cancel{1+\cos^2\alpha}}-\frac{(1+\sin^2\alpha)^{\cancel{2}}}{\cancel{1+\sin^2\alpha}}=1+\cos^2\alpha-1-\sin^2\alpha=\cos^2\alpha-\sin^2\alpha=\boxed{\cos 2\alpha}$$

$\tan\alpha=\frac{4}{3}$  $\rightarrow$ چون انتهای کمان در ربع سوم است، $\sin$ و $\cos$ باید منفی باشند $\Rightarrow \sin\alpha=-\frac{4}{5}\quad \cos=-\frac{3}{5}$

$\cot\alpha=\frac{3}{4}$

$$\underbrace{\sin\left(\frac{9\pi}{2}+\alpha\right)}_{\cos\alpha}\underbrace{\cos\left(\frac{7\pi}{2}-\alpha\right)}_{-\sin\alpha}-\underbrace{\tan\left(\alpha-\frac{3\pi}{2}\right)}_{-\cot\alpha}=\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)+\frac{3}{4}=\frac{-12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\boxed{\frac{27}{100}}$$ ۷

$$(3\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x)\xrightarrow{x=\frac{\pi}{12}}3\cos\left(\frac{\pi}{3}\right)+\sqrt{2}\sin\left(\frac{\pi}{12}\right)-\sqrt{2}\cos\left(\frac{\pi}{12}\right)\rightarrow$$

$$3\cos\frac{\pi}{3}+2\Big(\underbrace{\frac{\sqrt{2}}{2}}_{\sin\frac{\pi}{4}}\sin\frac{\pi}{12}-\underbrace{\frac{\sqrt{2}}{2}}_{\cos\frac{\pi}{4}}\cos\frac{\pi}{12}\Big)=3\cos\frac{\pi}{3}-2\Big(\underbrace{\cos\frac{\pi}{4}\cos\frac{\pi}{12}-\sin\frac{\pi}{4}\sin\frac{\pi}{12}}_{\cos\left(\frac{\pi}{4}+\frac{\pi}{12}\right)=\cos\frac{\pi}{3}}\Big)=3\cos\frac{\pi}{3}-2\cos\frac{\pi}{3}$$ ۸

$$=\cos\frac{\pi}{3}=\frac{1}{2}$$

$$\tan\left(\frac{\alpha}{2}\right)=\frac{1}{4}\rightarrow\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}\rightarrow\tan\alpha=\frac{2\times\frac{1}{4}}{1-\frac{1}{16}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}$$

$$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\rightarrow 1+\frac{64}{225}=\frac{1}{\cos^2\alpha}\rightarrow\frac{289}{225}=\frac{1}{\cos^2\alpha}\rightarrow\cos\alpha=\frac{15}{17}$$

$$\tan\alpha=\frac{\sin\alpha}{\cos\alpha}\rightarrow\frac{8}{15}=\frac{\sin\alpha}{\frac{15}{17}}\rightarrow\sin\alpha=\frac{8}{17}$$ ۹

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{8(17-15)}{15\times\cancel{17}}}{\frac{-7}{\cancel{17}}}=\frac{\frac{8\times2}{15}}{-7}=\boxed{-\frac{16}{105}}$$

$$\frac{\cot\alpha}{\sin\alpha}>0\rightarrow\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}>0\rightarrow\frac{\cos\alpha}{\sin^2\alpha}>0\rightarrow$$ چون $\sin^2\alpha$ همواره مثبت است پس

$\cos\alpha>0$ (I)

$2\sin\alpha<\sin 2\alpha\rightarrow\cancel{2}\sin\alpha<\cancel{2}\sin\alpha\cos\alpha\rightarrow\sin\alpha<\sin\alpha\cos\alpha\Rightarrow\sin\alpha-\sin\alpha\cos\alpha<0\rightarrow\sin\alpha(1-\cos\alpha)<0$

چون $1\geq\cos\alpha\geq-1$ پس $(1-\cos\alpha)$ همواره مثبت است بنابراین $\sin\alpha<0$ (II) ۱۰

(I), (II) $\rightarrow\cos\alpha>0\;,\;\sin\alpha<0\Rightarrow$ ربع چهارم

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