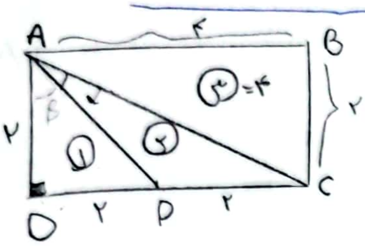
تحلیلی شماره ۲۷ — یازدهم دختر A — ۱۸ — دبیرستان

$\sqrt{3}$ ، 6 ، $\hat{\alpha}$ $S = 4.5$ بیشترین مقدار $\alpha = 120^\circ$ کمترین مقدار $\alpha = 60^\circ$

مساحت $= \dfrac{\sqrt{3} \times 6 \times \sin\hat{\alpha}}{2} = 4.5 \longrightarrow 3\sqrt{3} \times \sin\hat{\alpha} = 4.5 \longrightarrow \sin\hat{\alpha} = \dfrac{4.5}{3\sqrt{3}} = \dfrac{\sqrt{3}}{2}$ (۱)

$\Rightarrow \hat{\alpha} = 60^\circ$ یا $\hat{\alpha} = 120^\circ$ $\dfrac{120}{60} = 2$ برابر



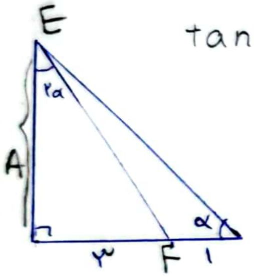
$AD^2 + DP^2 = AP^2 \longrightarrow \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} = AP$

$AC = \sqrt{4+16} = 2\sqrt{5}$ (۱)

① $S_{ADC} = 2 \times 2 \times \frac{1}{2} = 2$

$S_{APC} = \dfrac{2\sqrt{2} \times 2\sqrt{5} \times \sin\alpha}{2} = 2 \longrightarrow 4\sqrt{10} \times \sin\alpha = 4 \longrightarrow$

$\sin\alpha = \dfrac{1}{\sqrt{10}} \longrightarrow 1 + \cot^2 = \dfrac{1}{\sin^2} \longrightarrow \cot^2 = 10 - 1 = 9 \longrightarrow \cot\alpha = +3$

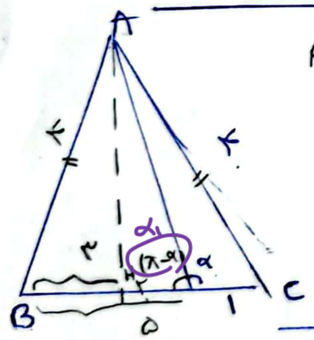


$\tan\alpha = \dfrac{A}{4}$, $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{2 \times \frac{A}{4}}{1 - \frac{A^2}{16}} = \dfrac{\frac{A}{2}}{\frac{16-A^2}{16}} \longrightarrow$ (۰/۵)

$\tan 2\alpha = \dfrac{1}{A} \longrightarrow \dfrac{8A}{16-A^2} = \dfrac{1}{A} \longrightarrow 8A^2 = 16 - A^2 \longrightarrow 9A^2 = 16$

$A = \dfrac{4}{3} \longrightarrow EF = \sqrt{1+(\frac{4}{3})^2} = \sqrt{1+\frac{16}{9}} = \dfrac{5}{3} \longrightarrow$

$\cot 2\alpha = \dfrac{A}{EF} = \dfrac{\frac{4}{3}}{\frac{5}{3}} = \dfrac{4}{5}$



$AH = \sqrt{7}$ فیثاغورس

$\tan(\pi - \alpha) = -\tan\alpha = \dfrac{\sqrt{7}}{2}$ (۱/۵)

$\tan\alpha = -\tan\alpha = \dfrac{-\sqrt{7}}{2}$

$2\sin^2 x + \cos^2 x = \dfrac{4}{3}$ $\tan^2 x = ?$

$\sin^2 x + \sin^2 x + \cos^2 x = \dfrac{4}{3} \longrightarrow \sin^2 x = \dfrac{1}{3} \longrightarrow \cos^2 x = 1 - \sin^2 x = 1 - \dfrac{1}{3}$ (۱)

$\cos^2 x = \dfrac{2}{3} \longrightarrow \tan^2 x = \dfrac{1}{\cos^2 x} - 1 \longrightarrow \dfrac{3}{2} - 1 = \dfrac{1}{2} = \tan^2 x$

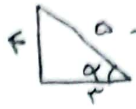
$\dfrac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \dfrac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} \Rightarrow \dfrac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2 - \sin^2\alpha} = \dfrac{(\sin^2\alpha - 2)^2}{2\sin^2\alpha}$ (۱)

$\dfrac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2 - \cos^2\alpha} = \dfrac{(\cos^2\alpha - 2)^2}{2 - \cos^2\alpha}$ $\Rightarrow (2 - \sin^2\alpha) - (2 - \cos^2\alpha) = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

۳) $\tan B_1 = \frac{AD}{AB} \to \tan 2\alpha = \frac{2}{x}$ $\quad \tan C = \frac{AB}{AC} \to \tan\alpha = \frac{x}{3}$

$\to \tan 2\alpha \to \frac{2}{x} = \frac{2\times\frac{x}{3}}{1-\frac{x^2}{9}} \to x = \frac{3}{2}$ $\quad \tan\alpha = \frac{1}{2}, \quad \cot\alpha = 2$

$\sin(\frac{9\pi}{2}+\alpha)\cos(\frac{7\pi}{2}-\alpha)-\tan(\alpha-\frac{3\pi}{2}) = ☆$



$\to \cos\alpha = \frac{-3}{5}$, $\sin\alpha = \frac{-4}{5}$

① $\sin(\frac{9\pi}{2}+\alpha) = \sin(\frac{\pi}{2}+\alpha) = \cos\alpha = -\frac{3}{5}$

② $\cos(\frac{7\pi}{2}-\alpha) = \cos(2\pi + \frac{3\pi}{2} - \alpha) = \cos(\frac{3\pi}{2}-\alpha) = -\sin\alpha = \frac{4}{5}$ (۵)

③ $\tan(\alpha-\frac{3\pi}{2}) = -\tan(\frac{3\pi}{2}-\alpha) = -\cot\alpha = -\frac{3}{4}$

$☆ \; (-\frac{3}{5})\times(\frac{4}{5}) + \frac{3}{4} = \frac{-48+75}{100} = \frac{27}{100}$

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۸) $3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x \quad x = \frac{\pi}{12}$

$3\cos 4\times\frac{\pi}{12} = \frac{3}{2}$, $\sqrt{2}\sin x - \sqrt{2}\cos x = \frac{\sqrt{2}}{2}(\sin x - \cos x)\times 2 =$ $\sin(x-\frac{\pi}{4})$ (۵)

$\Rightarrow 2\sin(\frac{\pi}{12}-\frac{\pi}{4}) = 2\times\sin(-\frac{\pi}{6}) = -1 \to \frac{3}{2} - 1 = \frac{1}{2}$

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۹) $\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = ?$ $\quad \tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} \to 1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \to$

$\tan\alpha = \frac{2\tan(\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})} = \frac{\frac{1}{2}}{1-\frac{1}{16}} = \frac{8}{15}$

$1 + \frac{64}{225} = \frac{1}{\cos^2\alpha} \to \cos^2\alpha = \frac{225}{289} \to \cos\alpha = \frac{15}{17} \Rightarrow \sin\alpha = \sqrt{1-\frac{225}{289}} = \frac{8}{17}$ (۵)

$\sin\alpha - \cos\alpha = \frac{8}{17} - \frac{15}{17} = \frac{-7}{17}$

$\tan\alpha - \sin\alpha = \frac{8}{15} - \frac{8}{17} = \frac{8\times 17 - 8\times 15}{15\times 17} = \frac{16}{255}$

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۱۰) $\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} = \frac{\cos\alpha}{\sin^2\alpha} > 0 \to \cos\alpha > 0$ ① $\longrightarrow$ α در ربع اول یا چهارم

($\sin^2\alpha > 0$) (۵)

ب) $2\sin\alpha < \sin 2\alpha \to 2\sin\alpha - 2\sin\alpha\cdot\cos\alpha < 0 \to \sin\alpha < 0$ ②

($\sin 2\alpha = 2\sin\alpha\cdot\cos\alpha$) $\quad 2\sin\alpha(1-\cos\alpha) < 0$, $(1-\cos\alpha) > 0$

①,② $\longrightarrow$ α در ربع چهارم