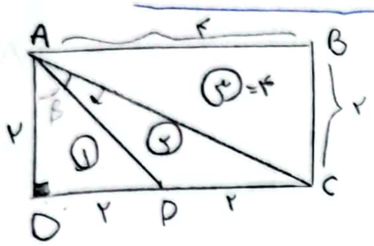
دیانا بابایی — یازدهم دختر A — تکلیف شماره ۲۷

$\sqrt{3}$ ، ۶ ، $\hat{\alpha}$ $\quad S = 4/5 \quad$ بیشترین مقدار α $= 120^\circ$ $\quad$ کمترین مقدار α $= 60^\circ$

مساحت $= \dfrac{\sqrt{3} \times 6 \times \sin\hat{\alpha}}{2} = 4/5 \longrightarrow 3\sqrt{3} \times \sin\hat{\alpha} = 4/5 \longrightarrow \sin\hat{\alpha} = \dfrac{4/5}{3\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \hat{\alpha} = 60^\circ$ یا $\hat{\alpha} = 120^\circ$ $\qquad \dfrac{120}{60} = 2$ برابر



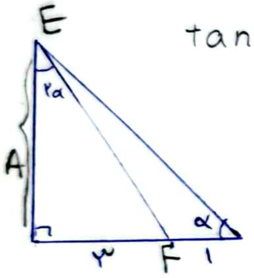
$AD^2 + DP^2 = AP^2 \to \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} = AP$

$AC = \sqrt{4+16} = 2\sqrt{5}$

① $S_{ADC} = 2 \times 2 \times \frac{1}{2} = 2$

$S_{APC} = \dfrac{2\sqrt{2} \times 2\sqrt{5} \times \sin\alpha}{2} = 2 \to 4\sqrt{10} \times \sin\alpha = 4 \to$

$\sin\alpha = \dfrac{1}{\sqrt{10}} \to 1 + \cot^2 = \dfrac{1}{\sin^2} \to \cot^2 = 10 - 1 = 9 \to \cot\alpha = +3$

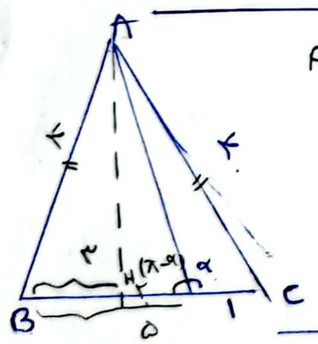


$\tan\alpha = \dfrac{A}{4} \quad , \tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha} = \dfrac{2 \times \frac{A}{4}}{1 - \frac{A^2}{16}} = \dfrac{\frac{A}{2}}{\frac{16 - A^2}{16}} \to$

$\tan 2\alpha = \dfrac{1}{A} \to \dfrac{8A}{16 - A^2} = \dfrac{1}{A} \to 8A^2 = 16 - A^2 \to 9A^2 = 16$

$A = \dfrac{4}{3} \to EF = \sqrt{1 + (\frac{4}{3})^2} = \sqrt{1 + \frac{16}{9}} = \dfrac{5}{3} \to$

$\cot 2\alpha = \dfrac{A}{EF} = \dfrac{\frac{4}{3}}{\frac{5}{3}} = \dfrac{4}{5}$



$AH = \sqrt{7}$ فیثاغورس

$\tan(\pi - \alpha) = -\tan\alpha = \dfrac{\sqrt{7}}{2}$

$2\sin^2 x + \cos^2 x = \dfrac{4}{3} \qquad \tan^2 x = ?$

$\sin^2 x + \sin^2 x + \cos^2 x = \dfrac{4}{3} \to \sin^2 x = \dfrac{1}{3} \to \cos^2 x = 1 - \sin^2 x = 1 - \dfrac{1}{3}$

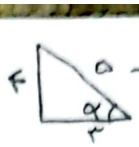
$\cos^2 x = \dfrac{2}{3} \to \tan^2 x = \dfrac{1}{\cos^2 x} - 1 \to \dfrac{3}{2} - 1 = \dfrac{1}{2} = \tan^2 x$

$\dfrac{\sin^4\alpha + 4\cos^2\alpha}{1 + \cos^2\alpha} - \dfrac{\cos^4\alpha + 4\sin^2\alpha}{1 + \sin^2\alpha} \Rightarrow \dfrac{\sin^4\alpha + 4 - 4\sin^2\alpha}{2 - \sin^2\alpha} = \dfrac{(\sin^2\alpha - 2)^2}{2 - \sin^2\alpha}$

$\dfrac{\cos^4\alpha + 4 - 4\cos^2\alpha}{2 - \cos^2\alpha} = \dfrac{(\cos^2\alpha - 2)^2}{2 - \cos^2\alpha}$

$\Rightarrow (2 - \sin^2\alpha) - (2 - \cos^2\alpha) = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$

$\sin(\frac{9\pi}{2}+\alpha)\cos(\frac{7\pi}{2}-\alpha)-\tan(\alpha-\frac{3\pi}{2})=$ ★ (1) (2) (3)



(1) $\sin(\frac{9\pi}{2}+\alpha)=\sin(\frac{\pi}{2}+\alpha)=\cos\alpha=-\frac{3}{5}$

(2) $\cos(\frac{7\pi}{2}-\alpha)=\cos(2\pi+\frac{3\pi}{2}-\alpha)=\cos(\frac{3\pi}{2}-\alpha)=-\sin\alpha=\frac{4}{5}$

(3) $\tan(\alpha-\frac{3\pi}{2})=-\tan(\frac{3\pi}{2}-\alpha)=-\cot\alpha=-\frac{3}{4}$

★ $(-\frac{3}{5})\times(\frac{4}{5})+\frac{3}{4}=\frac{-48+75}{100}=\frac{27}{100}$

$2\cos 4x+\sqrt{2}\sin x-\sqrt{2}\cos x$ $\quad x=\frac{\pi}{12}$

$2\cos 4\times\frac{\pi}{12}=2\cos\frac{\pi}{3}=\frac{2}{2}$, $\sqrt{2}\sin x-\sqrt{2}\cos x=\frac{\sqrt{2}}{2}(\sin x-\cos x)\times 2=$
$\sin(x-\frac{\pi}{4})$

$\Rightarrow 2\sin(\frac{\pi}{12}-\frac{\pi}{4})=2\times\sin(-\frac{\pi}{6})=-1\rightarrow\frac{2}{2}-1=\frac{1}{2}$

$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=?$ $\quad \tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}\rightarrow 1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\rightarrow$

$\tan\alpha=\frac{2\tan(\frac{\alpha}{2})}{1-\tan^2(\frac{\alpha}{2})}=\frac{\frac{1}{2}}{1-\frac{1}{4}}=\frac{4}{3}$

$1+\frac{9}{25}=\frac{1}{\cos^2\alpha}\rightarrow\cos^2\alpha=\frac{25}{34}\rightarrow\cos\alpha=\frac{5}{\sqrt{34}}\Rightarrow\sin\alpha=\sqrt{1-\frac{25}{34}}=\frac{3}{\sqrt{34}}$

$\sin\alpha-\cos\alpha=\frac{3}{\sqrt{34}}-\frac{5}{\sqrt{34}}=\frac{-2}{\sqrt{34}}$

$\tan\alpha-\sin\alpha=\frac{3}{5}-\frac{3}{\sqrt{34}}=\frac{3\times\sqrt{34}-3\times 5}{5\times\sqrt{34}}=\frac{-9}{10\sqrt{34}}$

$\frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha}=\frac{\cos\alpha}{\sin^2\alpha}>0\rightarrow\cos\alpha>0$ (1) ← ربع اول یا چهارم

$2\sin\alpha<\underbrace{\sin 2\alpha}_{2\sin\alpha\cdot\cos\alpha}\rightarrow 2\sin\alpha-2\sin\alpha\cdot\cos\alpha<0\rightarrow\sin\alpha<0$ (2)

$2\sin\alpha(1-\underbrace{\cos\alpha}_{>0})<0$
>0

(1),(2) ← ربع چهارم