ملیکا میرعقیلی جم

۱۹,۷۵

۱ج

$\cot\alpha = \frac{\cos\alpha}{\sqrt{\underbrace{1-\cos^2\alpha}_{\sin^2\alpha}}} \to \cot\alpha = \frac{\cos\alpha}{\underbrace{|\sin\alpha|}_{+\text{مثبت}}} \Rightarrow \begin{cases} \cos\alpha \geq 0 \Rightarrow \cot\alpha \geq 0 \Rightarrow \sin\alpha \geq 0 \\ \cos\alpha < 0 \Rightarrow \cot\alpha < 0 \Rightarrow \sin\alpha \geq 0 \end{cases}$ ①

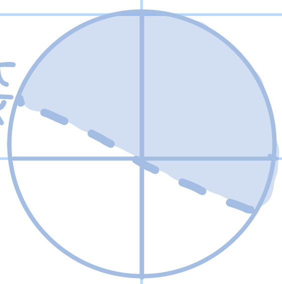
$\underbrace{\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha}}_{} = \frac{1-\sin\alpha}{|\cos\alpha|}$ ۵

$\begin{cases} \cos\alpha \geq 0 \to \frac{1-\sin\alpha}{\cos\alpha} \checkmark \\ \cos\alpha < 0 \Rightarrow \frac{1}{-\cos\alpha} - \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha - 1}{\cos\alpha} \times \end{cases} \Rightarrow$

$\cos\alpha \geq 0 \xrightarrow{①} \sin\alpha \geq 0 \Rightarrow$ $\boxed{\text{ربع اول}}$

۲ج

$-\frac{1}{2} < \sin 2x \leq 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1$



$\Rightarrow -2 < m-1 \leq 4 \Rightarrow -1 < m \leq 5$ ۵

$\Rightarrow \boxed{m \in (-1, 5]}$

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۳) $\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = -3 \to \sin x \cos x = \frac{-1}{3} = A$

$$\frac{1}{\sin^3 x+\cos^3 x}=\frac{1}{(\sin x+\cos x)(1-\sin x\cos x)}$$

$$A^2=\sin^2 x+\cos^2 x+2\sin x\cos x=\frac{1}{3}$$

$$\to A \left| \begin{array}{l} \frac{1}{\sqrt{3}} \ \times \\ -\frac{1}{\sqrt{3}} \ \checkmark \end{array} \right.$$

$$\to \frac{-9}{4\sqrt{3}}=-۰,۷۵\sqrt{3}$$

$$3\pi<4x<4\pi \Rightarrow \frac{3}{4}\pi<x<\pi \Rightarrow \cos x<0,\ \sin x>0$$

$$\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=-3 \Rightarrow \frac{\overbrace{\sin^2 x+\cos^2 x}^{1}}{\sin x\cos x}=-3 \Rightarrow -\frac{1}{3}=\sin x\cos x$$

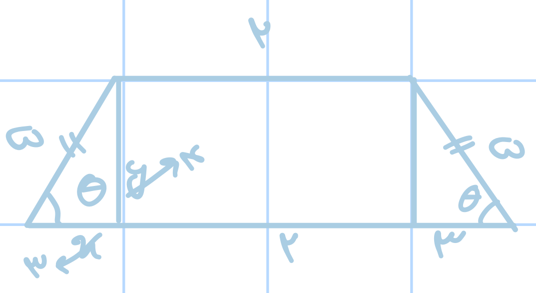
$$\to -\frac{1}{27}=\sin^3 x\cos^3 x,\quad \frac{1}{9}=\sin^2 x\cos^2 x$$

۱,۷۵

$$\frac{1}{\sin^3 x+\cos^3 x} \xrightarrow{\text{به توان ۲ میرسانیم}} \frac{1}{\sin^6 x+\cos^6 x+2\sin^3 x\cos^3 x}$$

$$\Rightarrow \frac{1}{\underbrace{(\sin^2 x+\cos^2 x)^3}_{1}-3\underbrace{\sin^2 x\cos^2 x}_{\frac{1}{9}}\underbrace{(\sin^2 x+\cos^2 x)}_{1}+2\underbrace{\sin^3 x\cos^3 x}_{-\frac{1}{27}}}$$

$$\Rightarrow \frac{1}{\underbrace{1-\frac{3}{9}-\frac{2}{27}}_{\frac{27-9-2}{27}=\frac{16}{27}}}=\frac{27}{16} \xrightarrow{\text{رادیکال میگیریم}} \boxed{\frac{3\sqrt{3}}{4}}$$



$$\Rightarrow 3+3+2=8$$

۵

$$\cos\theta=\frac{6}{10} \Rightarrow x=3,\ y=4$$

$$S=\frac{(2+8)}{2}\times 4=\boxed{20}$$

سوم

$\tan\left(\frac{3\pi}{2}+15\right)\times \sin(\pi+15) \rightarrow$

د

$-\cot(15)\times \tan(15) = -1$

$\sin(4\pi+15)\times\cos\left(\frac{3\pi}{2}-15\right) = \sin(15)\times -\sin(15)$

$= -\sin^2(15)$

$\rightarrow -1-(-\sin^2(15)) = -1+\sin^2(15) = -\cos^2 15$ و $\boxed{k=-1}$

دوم

$A = \sqrt{3}\underbrace{\cos(210)}_{-\frac{\sqrt{3}}{2}}\sin(243) - \sqrt{2}\underbrace{\sin(135)}_{\frac{\sqrt{2}}{2}}\cos(153)$

$\rightarrow A = \sqrt{3}\times -\frac{\sqrt{3}}{2}\times \sin(270-27) - \sqrt{2}\times\frac{\sqrt{2}}{2}\times\cos(180-27)$ د

$\sin(270-27) = \underbrace{\cancel{\cos 270 \sin 27}}_{0} + \underbrace{\sin 270}_{-1}\cos 27 = -\cos 27$

$\cos(180-27) = \underbrace{\cos 180}_{-1}\cos 27 + \underbrace{\cancel{\sin 180 \sin 27}}_{0} = -\cos 27$

$A = \underbrace{\sqrt{3}\times -\frac{\sqrt{3}}{2}\times -\cos 27}_{+\frac{3}{2}\cos 27} - \underbrace{\cancel{\sqrt{2}}\times\cancel{\frac{\sqrt{2}}{2}}\times -\cos 27^\circ}_{-\cos 27} = +\frac{3}{2}\cos 27 + \frac{2}{2}\cos 27$

$= \frac{5}{4}\cos 27 \rightarrow$ [illegible] $\boxed{\frac{5}{4}}$

۴۷

$$16\cos^2\left(\frac{\pi}{12}\right)\underbrace{\cos^2\left(\frac{\pi}{6}\right)}_{\frac{3}{4}}\underbrace{\cos^2\left(\frac{\pi}{3}\right)}_{\frac{1}{4}}\underbrace{\cos^2\left(\frac{2\pi}{3}\right)}_{\frac{1}{4}}$$

$$\cos^2\frac{\pi}{12} = \frac{1+\cos\pi/6}{2} \rightarrow \cos^2\pi/12 = \frac{2+\frac{\sqrt{3}}{2}}{2}$$

$$\cos^2\frac{\pi}{12} = \frac{2+\sqrt{3}}{4}$$

$$\rightarrow \cancel{16}\times\frac{(2+\sqrt{3})}{\cancel{4}}\times\frac{3}{\cancel{4}}\times\frac{1}{4}\times\frac{1}{4} = \boxed{\frac{6+3\sqrt{3}}{16}}$$

۴۸

$\cos x, \sin x < 0 \rightarrow$ ناحیه سوم

$$1-\sin x = 4+4\sin x \rightarrow \sin x = \frac{-3}{5} \rightarrow \cos x = \frac{-4}{5}$$

$$\tan^2\alpha/2 = \frac{1-\cos\alpha}{1+\cos\alpha} \rightarrow \tan^2\alpha/2 = \frac{\frac{9}{5}}{\frac{1}{5}} = 9 \rightarrow \pm 3$$

($\cos\alpha \rightarrow -4/5$)

ناحیه دوم

$\tan\alpha/_{۲} = -۳$

فرمول طلایی $\Rightarrow \sin^2\alpha = \dfrac{1-\cos 2\alpha}{2} \;,\; \cos^2\alpha = \dfrac{1+\cos 2\alpha}{2}$ س ۹

$\sin 2\alpha = 2\sin\alpha\cos\alpha \Rightarrow \sin\theta = 2\sin\theta/_{2}\cos\theta/_{2}$

$\dfrac{\sin\theta}{2\sin^2\theta/_{2}} + \dfrac{2\cos^2\theta/_{2}}{\sin\theta} \Rightarrow \dfrac{\sin^2\theta + \cancel{4\sin^2\theta/_{2}\cos^2\theta/_{2}}}{2\sin^2\theta/_{2}\,\sin\theta}$

$\Rightarrow \dfrac{\cancel{\sin^2\theta} + \sin^2\theta}{2\sin^2\theta/_{2} \times \cancel{\sin\theta}} = \dfrac{\cancel{2}\sin\theta}{\cancel{2}\sin^2\theta/_{2}} \Rightarrow \dfrac{\sin\theta}{\sin^2\theta/_{2}}$

$2\sin^2\theta$

$\Rightarrow \dfrac{2\cancel{\sin\theta/_{2}}\cos\theta/_{2}}{\sin^2\theta/_{2}} = \dfrac{2\cos\theta/_{2}}{\sin\theta/_{2}} = 2\cot\theta/_{2} \Rightarrow \boxed{k=2}$

$\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \dfrac{2}{100} + \cos^2\alpha = 1 \Rightarrow \cos\alpha = \pm\dfrac{\sqrt{98}}{10}$ س ۱۰

$\xrightarrow{\text{ناحیه سوم}} \cos\alpha = -\dfrac{7\sqrt{2}}{10}$

$\cos(\overbrace{\tfrac{11\pi}{4}}^{۳\pi/۴} + \alpha) = \underbrace{\cos\tfrac{11\pi}{4}}_{-\frac{\sqrt{2}}{2}}\;\overbrace{\cos\alpha}^{-\frac{7\sqrt{2}}{10}} - \underbrace{\sin\tfrac{11\pi}{4}}_{\frac{\sqrt{2}}{2}}\;\overbrace{\sin\alpha}^{\frac{\sqrt{2}}{10}} \longrightarrow$

$$-\frac{\sqrt{\cancel{۴}}}{\cancel{۴}} \times -\frac{۷\sqrt{\cancel{۴}}}{۱۰} - \frac{\sqrt{\cancel{۴}}}{\cancel{۴}} \times \frac{\sqrt{\cancel{۴}}}{۱۰} = \frac{۷}{۱۰} - \frac{۱}{۱۰} = \frac{۶}{۱۰} = \boxed{\frac{۳}{۵}}$$