حل مسائل تکمیلی ج

ج۱

$$\cot\alpha=\frac{\cos\alpha}{\underbrace{\sqrt{1-\cos^2\alpha}}_{\sin^2\alpha}}\rightarrow \cot\alpha=\frac{\cos\alpha}{\underbrace{|\sin\alpha|}_{+\text{همیشه}}}\Rightarrow \left.\begin{array}{l}\cos\alpha\geq0\Rightarrow\cot\alpha\geq0\Rightarrow\sin\alpha\geq0\\ \cos\alpha<0\Rightarrow\cot\alpha<0\Rightarrow\sin\alpha\geq0\end{array}\right\}(1)$$

$$\underbrace{\frac{1}{|\cos\alpha|}-\frac{\sin\alpha}{\cos\alpha}}=\frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\left.\begin{array}{l}\cos\alpha\geq0\rightarrow\dfrac{1-\sin\alpha}{\cos\alpha}\ \checkmark\\ \cos\alpha<0\Rightarrow\dfrac{1}{-\cos\alpha}-\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{\sin\alpha-1}{\cos\alpha}\ \times\end{array}\right\}\Rightarrow(2)$$

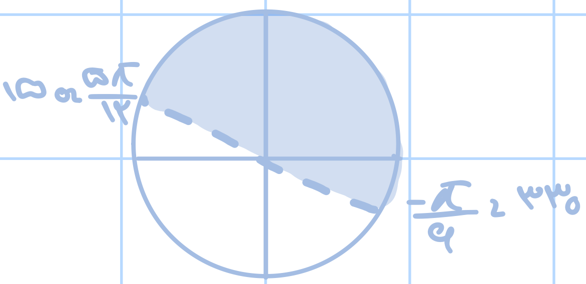
$$\cos\alpha\geq0\xrightarrow{(1)}\sin\alpha\geq0\Rightarrow\boxed{\text{ربع اول}}$$

ج۲

$$-\frac{1}{2}<\sin2x\leq1\Rightarrow-\frac{1}{2}<\frac{m-1}{4}\leq1$$

$$\Rightarrow-2<m-1\leq4\Rightarrow-1<m\leq5$$

$$\Rightarrow\boxed{m\in(-1,5]}$$



ج۳

$3\pi < 4x < 4\pi \Rightarrow \frac{3}{4}\pi < x < \pi \Rightarrow \cos x < 0 ,\ \sin x > 0$

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -3 \Rightarrow \frac{\overbrace{\sin^2 x + \cos^2 x}^{1}}{\sin x \cos x} = -3 \Rightarrow -\frac{1}{3} = \sin x \cos x$

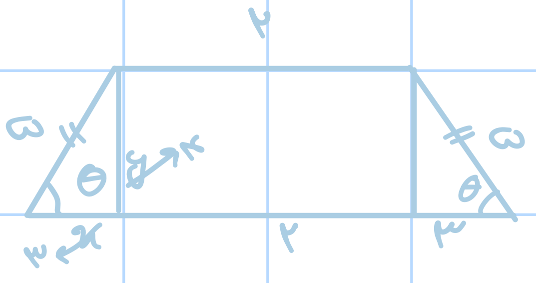
$\Rightarrow -\frac{1}{27} = \sin^3 x \cos^3 x ,\ \frac{1}{9} = \sin^2 x \cos^2 x$

$\frac{1}{\sin^3 x + \cos^3 x} \xrightarrow{\text{به توان ۲ می‌رسانیم}} \frac{1}{\sin^6 x + \cos^6 x + 2\sin^3 x \cos^3 x}$

$\Rightarrow \frac{1}{\underbrace{(\sin^2 x + \cos^2 x)^3}_{1} - 3\underbrace{\sin^2 x \cos^2 x}_{\frac{1}{9}}\underbrace{(\sin^2 x + \cos^2 x)}_{1} + 2\underbrace{\sin^3 x \cos^3 x}_{-\frac{1}{27}}}$

$\Rightarrow \frac{1}{\underbrace{1 - \frac{3}{9} - \frac{2}{27}}_{\frac{27-9-2}{27} = \frac{16}{27}}} = \frac{27}{16} \xrightarrow{\text{جذر می‌گیریم}} \boxed{\frac{3\sqrt{3}}{4}}$

ج۴



$\Rightarrow 3 + 3 + 2 = 8$

$\cos\theta = \frac{6}{10} \Rightarrow x = 3 ,\ y = 4$

$S = \frac{(2+8)}{2} \times 4 = \boxed{20}$

سوم

$$\tan\left(\frac{3\pi}{2}+15\right)\times\tan(\pi+15)\rightarrow$$

$$-\cot(15)\times\tan(15)=-1$$

$$\sin(4\pi+15)\times\cos\left(\frac{3\pi}{2}-15\right)=\sin(15)\times-\sin(15)$$

$$=-\sin^2(15)$$

$$\rightarrow -1-(-\sin^2(15))=-1+\sin^2(15)=-\cos^2 15 \Rightarrow \boxed{k=-1}$$

چهارم

$$A=\sqrt{3}\underbrace{\cos(210)}_{-\frac{\sqrt{3}}{2}}\sin(243)-\sqrt{2}\underbrace{\sin(135)}_{\frac{\sqrt{2}}{2}}\cos(153)$$

$$\Rightarrow A=\sqrt{3}\times-\frac{\sqrt{3}}{2}\times\sin(270-27)-\sqrt{2}\times\frac{\sqrt{2}}{2}\times\cos(180-27)$$

$$\sin(270-27)=\underbrace{\cos 270}_{0}\sin 27+\underbrace{\sin 270}_{-1}\cos 27=-\cos 27$$

$$\cos(180-27)=\underbrace{\cos 180}_{-1}\cos 27+\underbrace{\sin 180}_{0}\sin 27=-\cos 27$$

$$A=\underbrace{\sqrt{3}\times-\frac{\sqrt{3}}{2}\times-\cos 27}_{+\frac{3}{2}\cos 27}-\underbrace{\sqrt{2}\times\frac{\sqrt{2}}{2}\times-\cos 27}_{-\cos 27}=+\frac{3}{2}\cos 27+\frac{2}{2}\cos 27$$

$= \frac{5}{2}\cos 27 \rightarrow$ **پاسخ $\frac{5}{2}$**

۴۷

$$16\cos^2\left(\frac{\pi}{12}\right)\underbrace{\cos^2\left(\frac{\pi}{6}\right)}_{\frac{3}{4}}\underbrace{\cos^2\left(\frac{\pi}{3}\right)}_{\frac{1}{4}}\underbrace{\cos^2\left(\frac{2\pi}{3}\right)}_{\frac{1}{4}}$$

$$\cos^2\frac{\pi}{12} = \frac{1+\cos \pi/6}{2} \rightarrow \cos^2 \pi/12 = \frac{1+\frac{\sqrt{3}}{2}}{2}$$

$$\cos^2\frac{\pi}{12} = \frac{2+\sqrt{3}}{4}$$

$$\rightarrow 16 \times \frac{(2+\sqrt{3})}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} = \boxed{\frac{6+3\sqrt{3}}{16}}$$

۴۸

$\cos x, \sin x < 0 \rightarrow$ ربع سوم

$$1-\sin x = 4+4\sin x \rightarrow \sin x = -\frac{3}{5} \rightarrow \cos x = -\frac{4}{5}$$

$$\tan^2 \alpha/2 = \frac{1-\cos\alpha}{1+\cos\alpha} \rightarrow \tan^2 \alpha/2 = \frac{\frac{9}{5}}{\frac{1}{5}} = 9 \rightarrow \pm 3$$

[illegible]

tanα/۲ ، ۲-۳

۹) از فرمول طلایی ⇐ $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$, $\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha \Rightarrow \sin^2\theta = 4\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}$

$$\frac{\sin\theta}{2\sin^2\frac{\theta}{2}} + \frac{2\cos^2\frac{\theta}{2}}{\sin\theta} \Rightarrow \frac{\sin^2\theta + 4\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}\sin\theta}$$

$$\Rightarrow \frac{\sin^2\theta + \sin^2\theta}{2\sin^2\frac{\theta}{2}\times\sin\theta} = \frac{2\sin\theta}{2\sin^2\frac{\theta}{2}} \Rightarrow \frac{\sin\theta}{\sin^2\frac{\theta}{2}}$$

($\sin^2\theta + \sin^2\theta \to 2\sin^2\theta$)

$$\Rightarrow \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}} = \frac{2\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = 2\cot\frac{\theta}{2} \Rightarrow k=2$$

۱۰) $\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \frac{2}{100} + \cos^2\alpha = 1 \Rightarrow \cos\alpha = \pm\frac{\sqrt{98}}{10}$

ناحیه سوم ⇐ $\cos\alpha = -\frac{7\sqrt{2}}{10}$

$$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\frac{11\pi}{4}\cos\alpha - \sin\frac{11\pi}{4}\sin\alpha \longrightarrow$$

($\frac{11\pi}{4}$ → ۱۳۵°; $\cos\frac{11\pi}{4} = -\frac{\sqrt{2}}{2}$, $\cos\alpha = -\frac{7\sqrt{2}}{10}$, $\sin\frac{11\pi}{4} = \frac{\sqrt{2}}{2}$, $\sin\alpha = \frac{\sqrt{2}}{10}$)

$$-\frac{\sqrt{7}\sqrt{2}}{2} \times -\frac{\sqrt{7}\sqrt{2}}{10} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{10} = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$$