یازدهم دهم A — الف شماره — ۱۷,۲۵

(۱) $\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \Rightarrow \cot\alpha = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} \Rightarrow \cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha = |\sin\alpha| \Rightarrow \sin\alpha \geq 0$ ★

ناحیه اول

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{\cancel{1}}{|\cos\alpha|} - \frac{\cancel{1}+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \frac{+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$|\cos\alpha| = \cos\alpha \Rightarrow \cos\alpha > 0$ ★

(۲) $\sin 2x = \frac{m-1}{4},\ -\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x < 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} < 1$

$-2 < m-1 < 4 \Rightarrow -1 < m \leq 5$

۱,۷۵

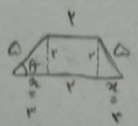
(۳) $\tan x + \cot x = -3,\ 3\pi < 4x < 4\pi \Rightarrow \frac{3\pi}{4} < x < \pi$

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -3 \Rightarrow \frac{\overbrace{\sin^2 x + \cos^2 x}^{1}}{\cos x \sin x} = -3 \Rightarrow -3\cos x \sin x = 1 \Rightarrow \cos x \sin x = -\frac{1}{3}$

$\sin^3 x + \cos^3 x = \underbrace{(\sin x + \cos x)}_{-\frac{1}{\sqrt{3}}}\underbrace{(\sin^2 x - \underbrace{\sin x\cos x}_{+\frac{1}{3}} + \cos^2 x)}_{\frac{3}{3}+\frac{1}{3}=\frac{4}{3}}$ $\quad \sin x + \cos x = \pm\sqrt{\underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{2\sin x\cos x}_{-\frac{2}{3}}} \Rightarrow \sin x + \cos x = -\frac{1}{\sqrt{3}}$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{-\frac{1}{\sqrt{3}} \times \frac{4}{3}} = -\frac{3\sqrt{3}}{4}$

(۴)



$\cos\theta = \frac{x}{5} = \frac{3}{5} \Rightarrow \frac{x}{5} = \frac{3}{5} \Rightarrow x = 3 \Rightarrow$ اندازه قاعده بزرگ $= 8$

مساحت مربع $= 2 \times 2 = 4$ $\quad$ مساحت مثلث $= \frac{3 \times 2}{2} = 3$

مساحت ذوزنقه $= 4 + 3 + 3 = 8$

$\cos\theta = \frac{x}{5} = \frac{6}{10}$

$\sin\theta = \frac{h}{5} = \frac{8}{10}$

$S = \frac{(2+8)}{2} \times 4 = 20$

۱

(۵) $\tan(285)\tan(-165) - \sin(1095)\cos(255) = -1 + \sin^2(15) = K\cos^2(15) \Rightarrow$

$\underbrace{\underbrace{\tan(270+15)}_{-\cot(15)}\ \underbrace{\tan(-180+15)}_{+\tan(15)}}_{-1}\quad \underbrace{\underbrace{\sin(1080+15)}_{+\sin(15)}\ \underbrace{\cos(270-15)}_{-\sin(15)}}_{-\sin^2(15)}$

$-\cos^2(15) = K\cos^2(15) \Rightarrow K = -1$

(۶) $A = \sqrt{3}\cos(210)\sin(243) - \sqrt{2}\sin(135)\cos(153) = +\frac{3}{2}\cos(27) + \cos(27) = \frac{5}{2}\cos(27) \Rightarrow$ برابر $\frac{5}{2}$

$\underbrace{\underbrace{\cos(270-60)}\ \underbrace{\sin(270-27)}}_{-\sqrt{3}\sin(60) \times -\cos(27)}$, $\quad -\sqrt{3}\sin(60) = -\frac{3}{2}$

$\underbrace{\sqrt{2}\sin(180-45)}_{\cancel{\sqrt{2}\sin(45)}}\ \underbrace{\cos(180-27)}_{-\cos(27)}$, $\quad \sqrt{2}\sin(45) = 1$

(۷) $f\left(\frac{\pi}{24}\right) = \underbrace{16\cos^2\left(\frac{\pi}{12}\right)}\ \underbrace{\cos^2\left(\frac{\pi}{6}\right)}\ \underbrace{\cos^2\left(\frac{\pi}{3}\right)}\ \cos^2\left(\frac{2\pi}{3}\right) =$

$16\left(\frac{1+\cos\frac{\pi}{6}}{2}\right)\left(\frac{1+\cos\frac{\pi}{3}}{2}\right)\left(\frac{1+\cos\frac{2\pi}{3}}{2}\right)\left(\frac{1+\cos\frac{4\pi}{3}}{2}\right)$

$\cancel{16}\left(\frac{1+\frac{\sqrt{3}}{2}}{\cancel{2}}\right)\left(\frac{1+\frac{1}{2}}{\cancel{2}}\right)\left(\frac{1-\frac{1}{2}}{\cancel{2}}\right)\left(\frac{1+\frac{1}{2}}{\cancel{2}}\right) = \underbrace{\left(1+\frac{\sqrt{3}}{2}\right)}_{\frac{2+\sqrt{3}}{2}}\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)\left(\frac{1}{2}\right) = \frac{18+9\sqrt{3}}{16}$

۰,۵

$\frac{1-\sin x}{1+\sin x} = ۴ \Rightarrow 1-\sin x = ۴ + ۴\sin x \Rightarrow ۵\sin x = -۳$

$\sin x = -\frac{۳}{۵}$

$\sin^۲ x + \cos^۲ x = ۱$

$\cos^۲ x = ۱ - \frac{۹}{۲۵}$

$\cos^۲ x = \frac{۱۶}{۲۵} \Rightarrow \cos x = -\frac{۴}{۵}$ ۸

$\tan\frac{x}{۲} = ? \Rightarrow \tan\frac{x}{۲} = \frac{\sin x}{1+\cos x} = \frac{-\frac{۳}{۵}}{1-\frac{۴}{۵}} = \frac{-\frac{۳}{۵}}{\frac{۱}{۵}} = -۳$

۵

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \Rightarrow \frac{\cancel{۲}\cancel{\sin\frac{\alpha}{۲}}\cos\frac{\alpha}{۲}}{\cancel{۲}\,\cancel{\sin^۲\frac{\alpha}{۲}}_{\sin\frac{\alpha}{۲}}} + \frac{\cancel{۲}\,\cancel{\cos^۲\frac{\alpha}{۲}}^{\cos\frac{\alpha}{۲}}}{\cancel{۲}\sin\frac{\alpha}{۲}\,\cancel{\cos\frac{\alpha}{۲}}} = \cot\frac{\alpha}{۲} + \cot\frac{\alpha}{۲} = ۲\cot\frac{\alpha}{۲} \Rightarrow k = ۲$ ۹

۵

$\sin\alpha = \frac{\sqrt{۲}}{۱۰}$, انتهای کمان α در ربع دوم دایره مثلثاتی است $\Rightarrow \cos\left(\frac{11\pi}{۴}+\alpha\right) = ? \Rightarrow \underbrace{-\frac{\sqrt{۲}}{۲}\times -\frac{۷\sqrt{۲}}{۱۰}}_{\frac{۷}{۱۰}} - \underbrace{\frac{\sqrt{۲}}{۲}\times\frac{\sqrt{۲}}{۱۰}}_{\frac{۱}{۱۰}} = \frac{۷}{۱۰} - \frac{۱}{۱۰} = \frac{۶}{۱۰} = \frac{۳}{۵}$ ۱۰

$\cos\left(\frac{11\pi}{۴}\right) \Rightarrow$ ربع دوم $= -\frac{\sqrt{۲}}{۲}$ $\quad \sin\alpha = \frac{\sqrt{۲}}{۱۰}$ $\quad 1 - \frac{۲}{۱۰۰} = \frac{۹۸}{۱۰۰} \Rightarrow \cos\alpha = \frac{\sqrt{۹۸}}{۱۰} = -\frac{۷\sqrt{۲}}{۱۰}$

$\sin\left(\frac{11\pi}{۴}\right) \Rightarrow$ " " $= +\frac{\sqrt{۲}}{۲}$ $\quad \cos\alpha = -\frac{۷\sqrt{۲}}{۱۰}$

۵

۱۱) $f\left(\frac{\pi}{۱۲}\right) = ۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\cos^۲\left(\frac{\pi}{۶}\right)\cos^۲\left(\frac{\pi}{۳}\right)\cos^۲\left(\frac{۲\pi}{۳}\right)$

$\cos^۲\frac{\pi}{۱۲} = \frac{1+\cos\frac{\pi}{۶}}{۲} = \frac{۲+\sqrt{۳}}{۴}$

$۱۶\left(\frac{۲+\sqrt{۳}}{۴}\right)\times\frac{۳}{۴}\times\frac{۱}{۴}\times\frac{۱}{۴} = \frac{۳(۲+\sqrt{۳})}{۱۶}$

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