یازدهم دهم A

کلید شماره

①

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \Rightarrow \cot\alpha = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} \Rightarrow \cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha = |\sin\alpha| \Rightarrow \sin\alpha \geq 0$ ★

ناحیه اول

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{\cancel{1}}{|\cos\alpha|} \; \frac{\cancel{-1}+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \frac{+\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$|\cos\alpha| = \cos\alpha \Rightarrow$

$\cos\alpha > 0$ ★

②

$\sin 2x = \frac{m-1}{4}, \; -\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x < 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} < 1$

$-2 < m-1 < 4 \Rightarrow -1 < m < 5$

③

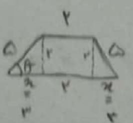
$\tan x + \cot x = -3, \; 3\pi < 4x < 4\pi \Rightarrow \frac{3\pi}{4} < x < \pi$

$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = -3 \Rightarrow \frac{\overbrace{\sin^2 x + \cos^2 x}^{1}}{\cos x \sin x} = -3 \Rightarrow -3\cos x \sin x = 1 \Rightarrow \cos x \sin x = -\frac{1}{3}$

$\sin^3 x + \cos^3 x = \underbrace{(\sin x + \cos x)}_{\frac{1}{\sqrt{3}}} \underbrace{(\overbrace{\sin^2 x - \sin x\cos x + \cos^2 x}^{1})}_{\frac{3}{3}+\frac{1}{3}=\frac{4}{3}} \quad \sin x + \cos x = \pm\sqrt{\underbrace{\sin^2 x + \cos^2 x}_{1} + \underbrace{2\sin x\cos x}_{-\frac{2}{3}}} \Rightarrow \sin x + \cos x = -\frac{1}{\sqrt{3}}$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{-\frac{1}{\sqrt{3}} \times \frac{4}{3}} = -\frac{3\sqrt{3}}{4}$

④



$\cos\theta = \frac{x}{5} = \frac{3}{5} \Rightarrow \frac{x}{5} = \frac{3}{5} \Rightarrow x = 3 \Rightarrow$ اندازه قاعده بزرگ $= 8$

مساحت مربع $= 2 \times 2 = 4$ مساحت مثلث $= \frac{3 \times 2}{2} = 3$

مساحت ذوزنقه $= 4 + 3 + 3 = 8$

⑤ ★

$\tan(285)\tan(-165) - \sin(1095)\cos(255) = -1 + \sin^2(15) = K\cos^2(15) \Rightarrow$

$\underbrace{\underbrace{\tan(270+15)}_{-\cot(15)} \; \underbrace{\tan(-180+15)}_{+\tan(15)}}_{-1} \; \underbrace{\underbrace{\sin(1080+15)}_{+\sin(15)} \; \underbrace{\cos(270-15)}_{-\sin(15)}}_{-\sin^2(15)}$

$-\cos^2(15) = K\cos^2(15) \Rightarrow K = -1$

⑥

$A = \sqrt{3}\cos(210)\sin(243) - \sqrt{2}\sin(135)\cos(153) = +\frac{3}{2}\cos(27) + \cos(27) = \frac{5}{2}\cos(27) \Rightarrow$ برابر $\frac{5}{2}$

$\underbrace{\underbrace{\cos(270-60)}_{-\sqrt{3}\sin(60)} \times \underbrace{\sin(270-27)}_{-\cos(27)}}_{-\frac{3}{2}} \qquad \underbrace{\underbrace{\sqrt{2}\sin(180-45)}_{\cancel{\sqrt{2}\sin(45)}}}_{1} \; \underbrace{\cos(180-27)}_{-\cos(27)}$

⑦

$f(\frac{\pi}{24}) = 16\cos^2(\frac{\pi}{12})\cos^2(\frac{\pi}{6})\cos^2(\frac{\pi}{3})\cos^2(\frac{2\pi}{3}) =$

$16(\frac{1+\cos\frac{\pi}{6}}{2})(\frac{1+\cos\frac{\pi}{3}}{2})(\frac{1+\cos\frac{2\pi}{3}}{2})(\frac{1+\cos\frac{4\pi}{3}}{2})$

$\cancel{16}(\frac{1+\frac{\sqrt{3}}{2}}{\cancel{2}})(\frac{1+\frac{1}{2}}{\cancel{2}})(\frac{1-\frac{1}{2}}{\cancel{2}})(\frac{1+\frac{1}{2}}{\cancel{2}}) = (\underbrace{1+\frac{\sqrt{3}}{2}}_{\frac{2+\sqrt{3}}{2}})(\frac{3}{2})(\frac{3}{2})(\frac{1}{2}) = \frac{18+9\sqrt{3}}{12}$

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$$\frac{1-\sin x}{1+\sin x} = 4 \Rightarrow 1-\sin x = 4 + 4\sin x \Rightarrow 5\sin x = -3$$ ۸

$$\sin x = -\frac{3}{5}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \frac{9}{25}$$

$$\cos^2 x = \frac{16}{25} \Rightarrow \cos x = -\frac{4}{5}$$

$$\tan\frac{x}{2} = ? \Rightarrow \tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{-\frac{3}{5}}{1-\frac{4}{5}} = \frac{-\frac{3}{5}}{\frac{1}{5}} = \underline{-3}$$

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} \Rightarrow \frac{\cancel{2\sin\frac{\alpha}{2}}\cos\frac{\alpha}{2}}{\cancel{2\sin^2\frac{\alpha}{2}}} + \frac{\cancel{2\cos^2\frac{\alpha}{2}}}{\cancel{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}} = \cot\frac{\alpha}{2} + \cot\frac{\alpha}{2} = 2\cot\frac{\alpha}{2} \Rightarrow k = 2$$ ۹

$$\cos\frac{\alpha}{2} \qquad \sin\frac{\alpha}{2}$$

$\sin\alpha = \frac{\sqrt{2}}{10}$ و انتهای کمان α در ربع دوم دایره مثلثاتی است $\Rightarrow \cos\left(\frac{11\pi}{4}+\alpha\right) = ? \Rightarrow \underbrace{-\frac{\sqrt{2}}{2}\times -\frac{7\sqrt{2}}{10}}_{\frac{7}{10}} - \underbrace{\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}}_{\frac{1}{10}} = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$ ۱۰

$\cos\left(\frac{11\pi}{4}\right) \Rightarrow$ (ربع دوم) $= -\frac{\sqrt{2}}{2}$ $\qquad \sin\alpha = \frac{\sqrt{2}}{10}$ $\qquad 1 - \frac{2}{100} = \frac{98}{100} \Rightarrow \cos\alpha = \frac{\sqrt{98}}{10} = -\frac{7\sqrt{2}}{10}$

$\sin\left(\frac{11\pi}{4}\right) \Rightarrow$ ″ $= +\frac{\sqrt{2}}{2}$ $\qquad \cos\alpha = -\frac{7\sqrt{2}}{10}$