نام/نام خانوادگی: پارسا شهرستانی — تکلیف شماره ۲۸ — دفتر A

1

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$

باتوجه به ① و ② در ناحیه اول است (جواب)

$\frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{\sin\alpha}{|\cos\alpha|} = \tan\alpha \to \cos\alpha > 0$ ①

$\cot = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \} \to \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \to \sin\alpha > 0$ ②

2

$\sin 2x = \frac{m-1}{4}$, $\frac{\pi}{12} < x < \frac{5\pi}{12} \xrightarrow{\times 2} -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \xrightarrow{\sin} -\frac{1}{2} < \sin 2x \le 1$

$\to -\frac{1}{2} < \frac{m-1}{4} \le 1 \to -2 < m-1 \le 4 \to -1 < m \le 5 \to (-1, 5]$ جواب

3

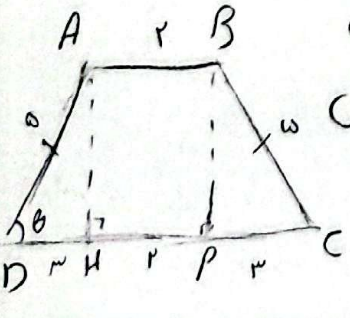
① $\tan x + \cot x = -3$, $3\pi < 4x < 4\pi \to$ ① $= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = -3 \to \sin x \cos x = -\frac{1}{3}$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 + \cos^2 - \sin x \cos x)} = \frac{1}{-\frac{1}{\sqrt{3}} \times \frac{4}{3}} = -\frac{3\sqrt{3}}{4}$ جواب ۱

② $(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x \cos x = 1 + 2(-\frac{1}{3}) = \frac{1}{3} \to \sin x + \cos x \begin{cases} \frac{1}{\sqrt{3}} \times \\ -\frac{1}{\sqrt{3}} \end{cases} \Rightarrow \sin x + \cos x = -\frac{1}{\sqrt{3}}$

③ $3\pi < 4x < 4\pi \xrightarrow{\div 4} \frac{3\pi}{4} < x < \pi \to$ ناحیه ۲ ، $|\sin x| < |\cos x|$

4



$\cos\theta = \frac{6}{10} \to \frac{DH}{AD} = \frac{6}{10} \to DH = 3 = PC$

فیثاغورس: $AH = \sqrt{25-9} = 4$

$S = \frac{(2+8) \times 4}{2} = 20$ جواب

5

$\tan(285)\tan(-165) - \sin(1095)\cos(255)$

$\tan(270+15)\tan(-180+15) - \sin(1080+15)\cos(270-15)$

$\to \underbrace{(-\cot 15)(\tan 15)}_{-1} - (\sin 15)(-\sin 15) = -1 + \sin^2 15 = -(1 - \sin^2 15)$

$= -\cos^2 15 = k\cos^2 15 \to k = -1$ جواب ۳

$A = \sqrt{3}\cos(210^\circ)\sin(243^\circ) - \sqrt{2}\sin(135)\cos(153^\circ) =$

$A = \sqrt{3}(\cos 180+30)\sin(270-27) - \sqrt{2}(\sin(90+45)\cos(180-27) =$

$= \sqrt{3}(-\cos 30)(-\cos 27) - \sqrt{2}(\sin 45)(-\cos 27) = \sqrt{3}(\frac{\sqrt{3}}{2})(\cos 27) + \sqrt{2}\frac{\sqrt{2}}{2}(\cos 27)$ 6

$= \frac{5}{2}(\cos 27) \Rightarrow \frac{\frac{5}{2}\cos 27}{\cos 27} = \frac{5}{2}$ برابر ← جواب

$F(x) = 16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)$ $\quad F(\frac{\pi}{36})$

$f(\frac{\pi}{36}) = 16\cos^2(\frac{\pi}{12})\cos^2(\frac{\pi}{6})\cos^2(\frac{\pi}{3})\cos^2(\frac{2\pi}{3})$

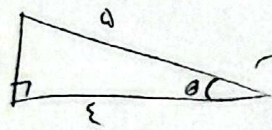
$16 \times \frac{\sqrt{3}+2}{4} \times \frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{6+3\sqrt{3}}{16}$ ⇐ جواب 7

$\cos^2(\frac{\pi}{12}) = \cos^2 15 = \frac{1+\cos 30}{2}$

$\cos^2 15 = (\frac{\sqrt{2+\sqrt{3}}}{2})^2 = \frac{2+\sqrt{3}}{4}$

$\frac{1-\sin x}{1+\sin x} = 4 \quad \tan\frac{x}{2} = ? \longrightarrow \tan\frac{x}{2} = \frac{\sin x}{1+\cos x} = \frac{-\frac{3}{5}}{1-\frac{4}{5}} = -3$ ⇐ جواب

$1-\sin x = 4+4\sin x \longrightarrow -3 = 5\sin x \to \sin x = -\frac{3}{5}$ 8



$\longrightarrow \cos x = \frac{4}{5} \xrightarrow{\text{ربع سوم}} -\frac{4}{5}$

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{2}$

① $\frac{\sin\theta}{1+\cos\theta} = \tan\frac{\theta}{2} \xrightarrow{\text{معکوس}} \frac{1+\cos\theta}{\sin\theta} = \cot\frac{\theta}{2}$

② $\frac{1-\cos\theta}{\sin\theta} = \tan\frac{\theta}{2} \xrightarrow{\text{معکوس}} \frac{\sin\theta}{1-\cos\theta} = \cot\frac{\theta}{2}$

$\xrightarrow{①,②} \cot\frac{\theta}{2} + \cot\frac{\theta}{2} = 2\cot\frac{\theta}{2}$ 9

$k = 2$ ⇐ جواب

$\cos(\frac{11\pi}{4}+\alpha) \quad \cos^2\alpha + \sin^2\alpha = 1 \to \cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{\frac{49}{50}} = -\frac{7}{5\sqrt{2}}$ (ربع ②)

$\cos(2\pi+\frac{3\pi}{4}+\alpha) = \cos(\frac{3\pi}{4}+\alpha) = \cos\alpha\cos\frac{3\pi}{4} - \sin\alpha\sin\frac{3\pi}{4}$ 10

$\Rightarrow (-\frac{7}{5\sqrt{2}} \times -\frac{\sqrt{2}}{2}) - (\frac{\sqrt{2}}{10} \times \frac{\sqrt{2}}{2}) = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$ ⇐ جواب