شماره ۲۸

ساینا هادیان

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(1)

$\frac{1}{\sqrt{\cos^2 x}} - \frac{1}{\cos x} = \frac{1-\sin x}{|\cos x|} \Rightarrow \frac{1}{|\cos x|} - \frac{\sin x}{\cos x} = \frac{1-\sin x}{|\cos x|} \Rightarrow \frac{\sin x}{\cos x} = \frac{\sin x}{|\cos x|}$ (1)

$\cot x = \frac{\cos x}{\sqrt{1-\cos^2 x}} \Rightarrow \frac{\cos x}{\sin x} = \frac{\cos x}{|\sin x|} \Rightarrow \sin x = |\sin x|$ (2) $\Rightarrow \cos x = |\cos x|$

$\Rightarrow \cos x, \sin x \geq$ **ربع اول**

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(2)

$-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x \leq 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1$

$\sin 2x = \frac{m-1}{4}$

$\Rightarrow -1 < m \leq 5$

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(3)

$\tan x + \cot x = \frac{2}{\sin 2x} = -3 \Rightarrow \sin 2x = -\frac{2}{3} \to \sin 2x = 2\sin x \cos x = -\frac{2}{3} \to \sin x \cos x = -\frac{1}{3}$

$\frac{3\pi}{2} < 2x < 2\pi \Rightarrow \frac{3\pi}{4} < x < \pi$ $\quad \sin x + \cos x = \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)} = \frac{1}{\sqrt{\frac{1}{3}} \times \frac{4}{3}} = \frac{3\sqrt{3}}{4}$

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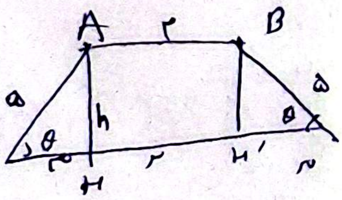
(4)



$\cos\theta = \frac{DH}{AD} = \frac{CH'}{BC} = \frac{6}{10} \Rightarrow DH = CH' = 3$

$AB = HH' = 2$

$\sin\theta = \sqrt{1 - 0/36} = 0/8 = \frac{h}{5} \Rightarrow h = 4$

$S = \frac{1}{2} \times 4 \times (2+8) = 20$

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(5)

$\tan(285)\tan(-165) - \sin(1095)\cos(255) = \tan(\frac{3\pi}{2}+15)\tan(\pi+15) - \sin(6\pi+15)\cos(\frac{3\pi}{2}-$

$\Rightarrow -\cot 15 \tan 15 + \sin 15(-\sin 15) = \sin^2 15 - 1 = -\cos^2 15 = k\cos^2 15 \Rightarrow k = -1$

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(6)

$\sqrt{3}\cos 210 \sin 240 - \sqrt{2}\sin 135 \cos 150 = \sqrt{48} - \frac{\sqrt{3}}{2}\sin(\frac{3\pi}{2} - 27) - \sqrt{2} \times \frac{\sqrt{2}}{2}\cos(\pi - 27)$

$= +\frac{\sqrt{3}}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27 \Rightarrow \frac{1/5}{2}$

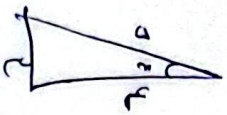
(۷)

$$F(x)=16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{4}\right)\cos^2\left(\frac{\pi}{3}\right)\frac{\sin^2\left(\frac{\pi}{2}+x\right)}{\sin^2 x}=16\times\frac{1}{4}\times\frac{3}{4}\times\frac{1}{2}\times\frac{1}{4}\cot^2 x$$

$$\frac{\sin^2\left(\frac{\pi}{2}+\frac{\pi}{8}\right)}{1-\cos\frac{\pi}{4}}=\frac{2\left(-\frac{\sqrt{2}}{2}\right)^2}{1-\cos\frac{\pi}{4}}=\frac{1}{1-\frac{\sqrt{2}}{2}}\times\frac{2}{2}=\frac{2}{2-\sqrt{2}}=2+\sqrt{2}$$

(۸)

$$\frac{1-\sin x}{1+\sin x}=\frac{1}{3}\Rightarrow 1-\sin x=\frac{1}{3}+\frac{1}{3}\sin x\Rightarrow\frac{2}{3}=\frac{4}{3}\sin x\Rightarrow\sin x=\frac{1}{2}$$



$$\cos x=-\frac{4}{5}$$

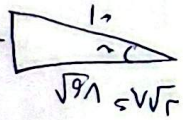
$$\tan\frac{x}{2}=\frac{\sin x}{1+\cos x}=\frac{\frac{3}{5}}{1-\frac{4}{5}}=3$$

(۹)

$$\frac{1-\cos\theta}{\sin\theta}=\frac{\sin\theta}{1+\cos\theta}=\tan\frac{\theta}{2}\longrightarrow\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{1}{\tan\frac{\theta}{2}}+\frac{1}{\tan\frac{\theta}{2}}=2\cot\frac{\theta}{2}=2\times 2=4$$

(۱۰)

$$\sin\alpha=\frac{\sqrt{2}}{10}$$



$$\cos\alpha=-\frac{7\sqrt{2}}{10}$$

$$\cos\left(\frac{11\pi}{4}+\alpha\right)=\cos\left(\frac{3\pi}{4}+\alpha\right)=\cos\frac{3\pi}{4}\cos\alpha-\sin\frac{3\pi}{4}\sin\alpha=\left(-\frac{\sqrt{2}}{2}\times-\frac{7\sqrt{2}}{10}\right)-\left(\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{10}\right)=\frac{7}{10}-\frac{1}{10}=\frac{6}{10}=\frac{3}{5}$$