یازدهم ریاضی A دهند — پاسخنامه تشریحی ۲۸ — آقا کرمانی

۲۰

۱)

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow |\sin\alpha| = \sin\alpha \to \sin\alpha > 0$ ناحیه اول یا دوم

$\frac{1-\sin\alpha}{|\cos\alpha|} = \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1}{|\cos\alpha|} - \frac{1}{\cot\alpha} = \frac{1}{|\cos|} - \frac{\sin\alpha}{\cos\alpha} \Rightarrow |\cos\alpha| = \cos\alpha$, $\cos\alpha > 0$ ناحیه اول یا چهارم

ناحیه اول

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۲)

$-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow \sin 2x = \frac{m-1}{4}$ $\quad -1 < m \leq 5 \Rightarrow m$

$-\frac{\pi}{6} < 2x < \frac{5\pi}{6}$ ($-30°$, $150°$) $\Rightarrow$ (دایره) $\Rightarrow -\frac{1}{2} < \sin 2x \leq 1 \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \Rightarrow -2 < m-1 \leq 4 \Rightarrow$

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۳)

$\tan\alpha + \cot\alpha = -3 \Rightarrow \tan\alpha + \frac{1}{\tan\alpha} + 3 = 0 \Rightarrow \tan^2\alpha + 3\tan\alpha + 1 = 0 \Rightarrow \tan\alpha$

$3\pi < 4x < 4\pi \Rightarrow \frac{3\pi}{4} < x < \pi \Rightarrow$ (دایره) $\quad (\sin x + \cos x)^2 = 2\sin x\cos x + \sin^2 x + \cos^2 x = \frac{1}{3} \Rightarrow -\frac{\sqrt{3}}{3}$

$\tan\alpha + \cot\alpha = \frac{2}{\sin 2\alpha} = -3 \Rightarrow \sin 2\alpha = \frac{2}{-3} = 2\sin\alpha\cos\alpha \Rightarrow \sin\alpha\cos\alpha = -\frac{1}{3} \Rightarrow$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x\cos x)} = \frac{1}{-\frac{\sqrt{3}}{3}\left(\frac{4}{3}\right)} = \frac{-9}{4\sqrt{3}}$

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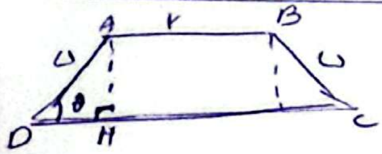
۴)



$\cos\theta = 0.6 = \frac{DH}{5} \Rightarrow DH = 3 \Rightarrow DC = 3 \times 2 + 2 = 8$

$AH = \sqrt{25 - 9} = 4$

مساحت $= \frac{(2+8)\times 4}{2} = 20$

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۵)

$\tan(285°)\tan(-165°) - \sin(1095°)\cos(255°) = k\cos^2 15$

$\tan\left(\frac{3\pi}{2}+15\right)\tan(-\pi+15°) - \sin(6\pi+15)\cos\left(\frac{3\pi}{2}-15\right) \Rightarrow$

$-\cot 15° \times \tan 15° - \sin 15(-\sin 15°) = -1 + \sin^2 15 = -\sin^2 15 - \cos^2 15 + \sin$

$\Rightarrow k = -1$

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۶)

$\sqrt{3}\cos(210)\sin(243) - \sqrt{2}\sin(135)\cos(153) = \frac{\sqrt{3}}{2}\times -\frac{\sqrt{3}}{2} \times -\cos 27 - \sqrt{2}\cdot\frac{\sqrt{2}}{2} - \cos$

$\sin(243) = \sin\left(\frac{3\pi}{2} - 27\right)$, $\cos(153) = \cos(\pi - 27)$

$+\cos 27\left(\frac{-3}{4} - 1\right)$

$\frac{-\frac{5}{4}}{\cos 27} = \frac{5}{4}$

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(۷)

$f\left(\frac{\pi}{24}\right) = 16\cos^2\left(\frac{\pi}{12}\right)\cos^2\left(\frac{\pi}{6}\right)\cos^2\left(\frac{\pi}{4}\right)\cos^2\left(\frac{3\pi}{4}\right)$

$16\times\left(\frac{2+\sqrt3}{4}\right)\times\frac{3}{4}\times\frac{1}{2}\times\frac{1}{2} = \frac{6+3\sqrt3}{16}$

$\cos^2 15 = \frac{1+\cos 30}{2} = \frac{1+\frac{\sqrt3}{2}}{2} = \frac{2+\sqrt3}{4}$

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(۸)

$\frac{1-\sin x}{1+\sin x} = 4 \Rightarrow 1-\sin x = 4+4\sin x \Rightarrow 5\sin x = -3,\ \sin x = -\frac{3}{5} \Rightarrow \cos x = -\frac{4}{5}$

$\tan^2\frac{x}{2} = \frac{1-\cos x}{1+\cos x} = \frac{\frac{9}{5}}{\frac{1}{5}} = 9 \Rightarrow \tan\frac{x}{2} = \pm 3 \xrightarrow{\text{ربع ۲ } \frac{x}{2}} -3$

ربع ۳ x ⟶ ربع ۲ $\frac{x}{2}$

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(۹)

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{2} \Rightarrow \frac{1}{\frac{\sin\theta}{1-\cos\theta}} = \frac{1}{\frac{1+\cos\theta}{\sin\theta}} = \tan\frac{\theta}{2} \Rightarrow$

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\cot\frac{\theta}{2} \Rightarrow k = 2$

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(۱۰)

$\cos\left(\frac{11\pi}{2}+\alpha\right) = \cos\left(\frac{3\pi}{2}+\alpha\right) = \cos\frac{3\pi}{2}\cos\alpha - \sin\frac{3\pi}{2}\sin\alpha =$

$\cos\alpha = \sqrt{1-\frac{2}{100}} = -\frac{7\sqrt2}{10}$ (ربع ۲)

$= \underbrace{\left(-\frac{\sqrt2}{2}\times -\frac{7\sqrt2}{10}\right)}_{\frac{7}{10}} - \underbrace{\left(\frac{\sqrt2}{2}\times\frac{\sqrt2}{10}\right)}_{\frac{1}{10}} = \frac{6}{10}$