① $\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow |\sin\alpha| = \sin\alpha \longrightarrow \sin\alpha > 0$ ناحیه اول یا دوم

$\frac{1-\sin\alpha}{|\cos\alpha|} = \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1}{|\cos\alpha|} - \frac{1}{\cot\alpha} = \frac{1}{|\cos|} - \frac{\sin\alpha}{\cos\alpha} \Rightarrow |\cos\alpha| = \cos\alpha$

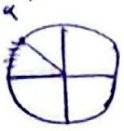
$\cos\alpha > 0$ ناحیه اول یا چهارم

ناحیه اول

② $-\frac{\pi}{11} < m < \frac{5\pi}{12} \Rightarrow \sin 4x = \frac{m-1}{2}$ $\quad -1 < m \leq 6 \Rightarrow m$

$\frac{-\pi}{9} < 2m < \frac{6\pi}{9} \Rightarrow$  $\Rightarrow -\frac{1}{2} \leq \sin^2 m \leq 1 \Rightarrow \frac{-1}{2} \leq \frac{m-1}{2} \leq 1 \Rightarrow -2 \leq m-1 \leq 2 \Rightarrow$

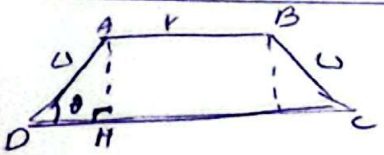
$\tan\alpha + \cot\alpha = -2 \Rightarrow \tan\alpha + \frac{1}{\tan\alpha} = -2 \Rightarrow \tan^2\alpha + 2\tan\alpha + 1 = 0 \Rightarrow \tan\alpha$

$2\pi < 2m < 4\pi \Rightarrow \frac{\pi}{2} < m < \pi \Rightarrow$ 

$(\sin\alpha\cos\alpha)^2 = \frac{1}{4}\sin^2 2\alpha = \frac{1}{4}(1-\cos^2 2\alpha) = \frac{1}{4}\left(1-\frac{3}{4}\right) = \frac{1}{16} \Rightarrow \sin\alpha\cos\alpha = \frac{1}{4}$

$\tan\alpha + \cot\alpha = \frac{2}{\sin 2\alpha} = -2 \Rightarrow \sin 2\alpha = -1 \Rightarrow 2\sin\alpha\cos\alpha = -1 \Rightarrow \sin\alpha\cos\alpha = -\frac{1}{2}$

$\frac{1}{\sin^2 m\cos^2 m} \cdot \frac{1}{(\sin m\cos m)(\sin^2 m + \cos^2 m - 2\sin m\cos m)} = \frac{1}{-\frac{1}{16}\left(\frac{3}{4}\right)} = \frac{-9}{4\sqrt{3}}$

④ 

$\cos\theta = \frac{9}{10} = \frac{DH}{10} \Rightarrow DH = 8 \Rightarrow DC = \sqrt{8^2+2^2} = 8$

$AH = \sqrt{100-64} = 6$

$S_{ABCD} = \frac{(2+8)}{2} \times 6 = 30$

⑤ $\tan(2\pi\omega)\tan(-19\omega^\circ) - \sin(10\omega)\cos(2\omega^\circ) = k\cos^2 10\omega$

$\tan\left(\frac{2\pi}{2}+10\omega\right)\tan(-\pi+10\omega^\circ) - \sin(4\pi+10\omega)\cos\left(\frac{10\pi}{2}-10\omega\right) \Rightarrow$

$-\cot 10\omega^\circ \times \tan 10\omega^\circ - \sin 10\omega(-\sin 10\omega) = -1 + \sin^2 10\omega = -\sin^2 10\omega - \cos^2 10\omega + \sin^2 10\omega$

$\Rightarrow k = -1$

$\sqrt{2}\cos(210^\circ)\sin(225^\circ) - \sqrt{2}\sin(120^\circ)\cos(150^\circ) = \frac{\sqrt{2}}{2}\alpha - \frac{\sqrt{2}}{2} = -\cos 45^\circ - \sqrt{2}\cdot\frac{\sqrt{2}}{2} = -\cos$

$\sin\left(\frac{5\pi}{2}-45^\circ\right) \qquad \cos(\pi-\pi)$

$+\cos 45^\circ\left(\frac{-\sqrt{3}}{2}-1\right)$

$\frac{-\frac{\omega}{2}}{\cos 45^\circ} = \frac{\omega}{2}$

(۷)

$$f\left(\frac{\pi}{۲۹}\right) \cdot ۱۹\cos^{۲}\left(\frac{\pi}{۱۲}\right)\cos^{۲}\left(\frac{\pi}{۹}\right)\cos^{۲}\left(\frac{۲\pi}{۳}\right)\cos^{۲}\left(\frac{۳\pi}{۲}\right)$$

$$۱۹\times\left(\frac{۳\sqrt{۳}}{۴}\right)\times\frac{۳}{۴}\times\frac{۱}{۴}\times\frac{۱}{۴} = \frac{۶۷+۳\sqrt{۳}}{۱۹}$$

$$\cos^{۲}۱۵^{\circ} = \frac{۱+\cos ۳۰^{\circ}}{۲} = \frac{۱+\frac{\sqrt{۳}}{۲}}{۲} = \frac{۲+\sqrt{۳}}{۴}$$

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(۸)

$$\frac{۱-\sin x}{۱+\sin x} = ۴ \Rightarrow ۱-\sin x = ۴+۴\sin x \Rightarrow ۵\sin x = -۳ \Rightarrow \sin x = -\frac{۳}{۵} \Rightarrow \cos x = -\frac{۴}{۵}$$

$$\tan^{۲}\frac{x}{۲} = \frac{۱-\cos x}{۱+\cos x} = \frac{\frac{۹}{۵}}{\frac{۱}{۵}} = ۹ \Rightarrow \tan\frac{x}{۲} = \pm ۳ \xrightarrow{\left(\frac{\pi}{۲}, \pi\right)} -۳$$

$\sin x \longrightarrow \sin\frac{x}{۲}$

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(۹)

$$\frac{\sin\theta}{۱-\cos\theta} + \frac{۱+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{۱} \Rightarrow \frac{۱}{\frac{\sin\theta}{۱-\cos\theta}} = \frac{۱}{\frac{۱+\cos\theta}{\sin\theta}} = \frac{۱}{\tan\frac{\theta}{۲}} \Rightarrow$$

$$\frac{\sin\theta}{۱-\cos\theta} + \frac{۱+\cos\theta}{\sin\theta} = ۲\cot\frac{\theta}{۲} \Rightarrow k = ۲$$

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(۱۰)

$$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right) = \cos\left(\frac{۳\pi}{۴}+\alpha\right) = \cos\frac{۳\pi}{۴}\cos\alpha - \sin\frac{۳\pi}{۴}\sin\alpha =$$

$\cos\alpha \xrightarrow{(\frac{\pi}{۲}, \pi)} \sqrt{۱-\frac{۹}{۱۰۰}} = -\frac{۷\sqrt{۲}}{۱۰}$

$$= \underbrace{\left(-\frac{\sqrt{۲}}{۲}\times-\frac{۷\sqrt{۲}}{۱۰}\right)}_{\frac{۷}{۱۰}} - \underbrace{\left(\frac{\sqrt{۲}}{۲}\times\frac{\sqrt{۲}}{۱۰}\right)}_{\frac{۱}{۱۰}} = \frac{۶}{۱۰}$$