تالیف شماره‌ی ۲۸

1. $\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}$ $\qquad \frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$

$\frac{\cos}{\sin} = \frac{\cos}{\sqrt{\sin^2}} \Rightarrow \sqrt{\sin^2} = \sin \Rightarrow \sin > 0.$

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\frac{\cos\alpha}{\sin\alpha}} = \frac{1-\sin\alpha}{|\cos\alpha|}$ } نصب اول

$\hookrightarrow \frac{1}{\sqrt{\cos^2\alpha}} - \frac{\sin\alpha}{\cos\alpha} \Rightarrow \sqrt{\cos^2\alpha} = \cos\alpha \Rightarrow \cos\alpha > 0.$

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2. $\sin^2 x = \frac{m-1}{4}$ $\qquad -\frac{\pi}{12} < x < \frac{5\pi}{12}$

$\downarrow$

$-\frac{\pi}{6} < 2x < \frac{5\pi}{6} \rightarrow -\frac{1}{2} < \sin 2x \leq 1$

$\rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \rightarrow -1 < m \leq 5 \Rightarrow (-1, 5]$

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3. $\tan x + \cot x = -4$ $\qquad \pi < 4x < 4\pi$

$\frac{1}{\sin^2 x + \cos^2 x}$ $\qquad \searrow \tan + \cot = \frac{1}{\sin\cdot\cos} = -4 \Rightarrow \sin\cdot\cos = -\frac{1}{4}$

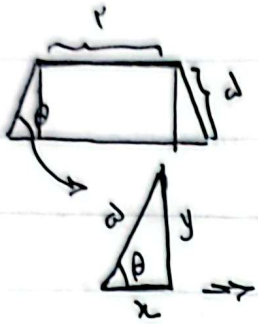
$\hookrightarrow$ صل و تجان $\rightarrow (\cos + \sin)(\sin^2 + \cos^2 - \sin\cdot\cos) \Rightarrow$ حاصل‌ی $= -\frac{3\sqrt{3}}{4}$

$\downarrow$

$\sqrt{1 + 2\sin\cdot\cos} = \sqrt{1 + -\frac{2}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{\sqrt{4}}$

احمد مانی

4.



محیط ذوزنقه $= ۲+۳+۳=۸$

مساحت $= \frac{(۳+۸)\times ۴}{۲} = ۲۰$

$\cos = \frac{x}{۵} = ۰,۹ \rightarrow x=۳$

$\hookrightarrow y = ۴$

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5. $\tan(۲۸۵)\tan(-۱۴۵) - \sin(۱۰۹۵)\cos(۲۵۵) = k\cos^۲ ۱۵$

$\tan(۲۸۵) = \tan(۲۸۵ - ۲\pi) = \tan(-۷۵) = -\tan(۷۵)$

$\tan(-۱۴۵) = -\tan(۱۴۵) = -(-\tan ۱۵) = \tan(۱۵)$

$\sin(۱۰۹۵) = \sin(۱۰۹۵ - ۴\pi) = \sin(۳۷۵) = \sin(۳۷۵ - ۲\pi) = \sin(۱۵)$

$\cos(۲۵۵) = \cos(۲۵۵ - ۲\pi) = \cos(-۱۰۵) = -\sin(۱۵)$

$\hookrightarrow \underbrace{-\tan(۷۵)\tan(۱۵)}_{-۱} - (-\sin(-۱۵)\sin(-۱۵)) = -۱ + \sin^۲ ۱۵$

$= -۱ + ۱ - \cos^۲ ۱۵$

$= -\cos^۲ ۱۵ \Rightarrow \boxed{k=-۱}$

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6. $A = \sqrt{۳}\cos(۲۱۰)\sin(۲۴۲) - \sqrt{۲}\sin(۱۳۵)\cos(۱۵۳)$

$\hookrightarrow -\frac{۳}{۲}\times(-\cos ۲۷) + \cos ۲۷ = \frac{۵}{۲}\cos ۲۷$

$\frac{A}{\cos ۲۷} \Rightarrow \frac{۵/۲\cos ۲۷}{\cos ۲۷} =$ برابر $۲/۵$

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7. $f(x) = ۱۶\cos^۲(۳x)\cos^۲(۹x)\cos^۲(۱۲x)\cos^۲(۲۴x)$

$f\left(\frac{\pi}{۲۴}\right) = ۱۶\cos^۲\left(\frac{\pi}{۸}\right)\cos^۲\left(\frac{\pi}{۴}\right)\cos^۲\left(\frac{\pi}{۳}\right)\cos^۲\left(\frac{۲\pi}{۳}\right)$

$= ۱۶\cos^۲(۱۵)\cos^۲(۴۵)\cos^۲(۶۰)\cos^۲(۱۲۰)$

$\left(\frac{\sqrt{۲}}{۲}\right)^۲ \quad \left(\frac{۱}{۲}\right)^۲ \quad \left(-\frac{۱}{۲}\right)^۲$

$= \frac{۳}{۴}\cos^۲(۱۵) \rightarrow \frac{۳}{۴}\left(\frac{۱+\cos ۳۰}{۲}\right) = \frac{۹+۳\sqrt{۳}}{۱۶}$

8. $\frac{1-\sin x}{1+\sin x} = 4 \hookrightarrow 1-\sin x = 4+4\sin x \rightarrow 5\sin x = -3 \rightarrow \sin = -\frac{3}{5}$

$\sin^2 + \cos^2 = 1 \rightarrow \frac{9}{25} + \cos^2 = 1 \rightarrow \cos^2 = \frac{16}{25} \rightarrow \cos = -\frac{4}{5}$ (رابع سوم)

$\tan x = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4}$

$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \rightarrow \frac{3}{4} = \frac{2z}{1-z^2} \rightarrow 3-3z^2 = 8z$ ($\tan\frac{x}{2} = z$)

$3z^2 + 8z - 3 = 0 \rightarrow z = -3$ ، $z = \frac{1}{3}$

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9. $\frac{2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2} + \frac{2\cos^2\theta/2}{2\sin\theta/2\cos\theta/2} = \cot\theta/2 + \cot\theta/2 = 2\cot\frac{\theta}{2} \rightarrow k = 2$

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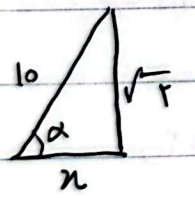
10. $\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\left(3\pi - \frac{\pi}{4} + \alpha\right)$

$\hookrightarrow -\cos\left(\alpha - \frac{\pi}{4}\right) = -\left(\cos\alpha\cos\frac{\pi}{4} + \sin\alpha\sin\frac{\pi}{4}\right)$

$= -\frac{\sqrt{2}}{2}(\cos\alpha + \sin\alpha) \rightarrow \cos\left(\frac{11\pi}{4}+\alpha\right)$

$\hookrightarrow -\frac{\sqrt{2}}{2}\left(-\frac{7\sqrt{2}}{10} + \frac{\sqrt{2}}{10}\right) = \frac{3}{5}$



$x = \sqrt{98} \rightarrow \cos\alpha = -7\sqrt{2}/10$