دیناتابری ۱/۵ یازدهم دفتر ۹ تکلیف شماره ۲۸

۱)

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \xrightarrow{(*)} \frac{1}{|\cos\alpha|} - \frac{|\sin\alpha|}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \quad (*)$$

$$\sqrt{\sin^2\alpha} = |\sin\alpha|$$

چون $|\sin\alpha| = \sin\alpha \leftarrow \sin\alpha > 0$

چون $|\cos\alpha| = \cos\alpha \leftarrow \cos\alpha > 0$ ← α در ربع اول

۲)

$$-\frac{\pi}{12} < x < \frac{5\pi}{12}, \quad \sin 2x = \frac{m-1}{4} \qquad m = ?$$

$$-\frac{1}{2} < \sin 2\alpha < 1 \to -\frac{1}{2} < \frac{m-1}{4} < 1 \to m \in (-1, 5)$$

۰/۵

$$-\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow \sin(-\frac{\pi}{6}) < \sin 2x < \sin(\pi - \frac{\pi}{6}) \to -\frac{1}{2} < \sin 2x < \frac{1}{2} \Rightarrow$$

$$-\frac{1}{2} < \frac{m-1}{4} < \frac{1}{2} \to \left(\frac{m-1}{4}\right) < \left(\frac{1}{2}\right) \xrightarrow{\times 4} m-1 < 2 \to m < 3$$

$$\left(\frac{m-1}{4}\right) > \left(-\frac{1}{2}\right) \xrightarrow{\times 4} m-1 > -2 \to m > -1$$

$$-1 < m < 3$$

۳)

$$\tan\alpha + \cot\alpha = -3, \quad 3\pi < 4x < 4\pi \to \frac{3\pi}{4} < x < \pi \to$$

۱

$$\sin\frac{\pi}{4} < \sin x < \sin\pi \to \frac{\sqrt{2}}{2} < \sin x < 0$$

$$-\cos\frac{\pi}{4} < \cos x < \cos\pi, \quad -\frac{\sqrt{2}}{2} < \cos x < -1 \to -1 < \sin x + \cos x < 0 \quad *$$

$$\frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha\cdot\cos\alpha} = \frac{1}{\sin\alpha\cdot\cos\alpha} = -3 \to \sin\alpha\cdot\cos\alpha = \frac{-1}{3} \quad (**)$$

$$\sin\alpha = \alpha, \ \cos\alpha = \beta \to \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) \to$$

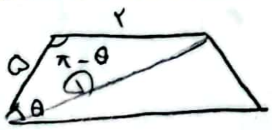
$$\sin^3\alpha + \cos^3\alpha = (\sin x + \cos x)^3 - 3\sin x\cdot\cos x(\sin x + \cos x) \quad (*),(**),(***)$$

$$\left(-\frac{1}{\sqrt{3}}\right)^3 - 3\left(-\frac{1}{3}\right)\left(-\frac{1}{\sqrt{3}}\right) = \frac{-1}{\sqrt{3}}^3 - \frac{1}{\sqrt{3}} = \frac{-4}{3\sqrt{3}}$$

$$(\sin\alpha + \cos\alpha)^2 = \sin^2\alpha + \cos^2\alpha + 2\underbrace{\sin\alpha\cdot\cos\alpha}_{-1/3} \to (\sin\alpha + \cos\alpha)^2 = \frac{1}{3} \xrightarrow{\sin\alpha+\cos\alpha<0} \quad (*)$$

$$\sin\alpha + \cos\alpha = \frac{-1}{\sqrt{3}} \quad (***)$$

۴)



$$\cos\theta = 0/6, \quad \cos(\pi-\theta) = -\cos\theta = -0/6$$

$$\sin(\underbrace{\pi-\theta}_{\alpha}) \Rightarrow \sin^2\alpha = 1 - \cos^2\alpha = 1 - \frac{9}{25} = \frac{16}{25} \to \sin\alpha = \frac{4}{5}$$

۰/۵

$$S_{①} = \frac{1}{2}\times 2\times 5\times\frac{4}{5} = 4 \longrightarrow S_{(2)} = 2\times 4 = 8$$

(۵

$\tan(۲۸۵^\circ)\cdot\tan(-۱۶۵) - \sin(۱۰۹۵)\cos(۲۵۵)$

$\tan(\cancel{۳۶۰}-۷۵)\times\underbrace{\tan(-(۱۸۰-۱۵))}_{-\tan(۱۸۰-۱۵)} - \sin(۳\times\cancel{۳۶۰}+۱۵)\cdot\cos(۱۸۰+۷۵) =$

$\underbrace{-\tan۷۵\times\tan۱۵}_{-۱} \underbrace{-\sin(۱۵)\times -\cos(۷۵)}_{\sin^۲ ۱۵} = -۱+\sin^۲ ۱۵$

$\underbrace{\cancel{-۱+\sin^۲ ۱۵}}_{\cos^۲ ۱۵} = k\cos^۲ ۱۵ \longrightarrow k = ۱$

(۶

$A = \sqrt{۳}\cos(۲۱۰)\sin(۲۴۳) - \sqrt{۲}\sin(۱۳۵)\cos۱۵۳$

$A = \sqrt{۳}\overbrace{\cos(\cancel{۱۸۰}+۳۰)}^{-\cos۳۰}\cdot\sin۶۳ - \sqrt{۲}\overbrace{\sin(\cancel{۱۸۰}-۴۵)}^{\sin۴۵}\cdot\cos(۱۸۰-۲۷)$

$-\sqrt{۳}\times\frac{۱}{۲}\times\sin۶۳ - \sqrt{۲}\times\frac{\sqrt{۲}}{۲}\times -\cos۲۷ = -\frac{\sqrt{۳}}{۲}\underbrace{\cancel{\sin۶۳}}_{\cos۲۷} + \cos۲۷ =$

$\cos۲۷\left(-\frac{\sqrt{۳}}{۲}+۱\right) = \left(\frac{۲-\sqrt{۳}}{۲}\right)\cos۲۷$

۷

$f(x) = ۱۶\cos^۲(۳x)\cos^۲(۶x)\cos^۲(۱۲x)\cos^۲(۲۴x) \qquad \frac{\pi}{۳۶} = ۵$

$f(\frac{\pi}{۳۶}) = f(۵) = ۱۶\cos^۲(۱۵)\times\underbrace{\cos^۲(\cancel{۹۰})}_{۰}\cos^۲(۱۸۰)\cos(۱۲۰) = ۰$

(۸

$\frac{۱-\sin x}{۱+\sin x} = ۴ \qquad \tan\frac{x}{۲} = ?$

$۴+۴\sin x = ۱-\sin x \longrightarrow ۵\sin x = -۳ \longrightarrow \sin x = \frac{-۳}{۵} , \cos^۲ x = \frac{۱۶}{۲۵} \longrightarrow$

$\cos x = -\frac{۴}{۵}\Big\} \quad \tan x = \frac{۳}{۴} \qquad \frac{x}{۲} = \alpha \longrightarrow x = ۲\alpha \Rightarrow$

$\tan۲\alpha = \frac{۲\tan\alpha}{۱-\tan^۲\alpha} \longrightarrow \tan x = \frac{۲\tan x/۲}{۱-\tan^۲ x/۲} \Rightarrow \frac{۳}{۴} = \frac{۲\tan x/۲}{۱-\tan^۲ x/۲} \longrightarrow$

$۳-۳\tan^۲ x/۲ = ۸\tan x/۲ \longrightarrow ۳\tan^۲ x/۲ + ۸\tan x/۲ - ۳ = ۰ \longrightarrow$

$\tan x/۲ = t \longrightarrow ۳t^۲+۸t-۳=۰ \longrightarrow \Delta = ۱۰۰ \longrightarrow \begin{cases} t_۱ = \frac{-۸+۱۰}{۶} = \frac{۲}{۶} = \frac{۱}{۳} & \text{ق غ} \\ t_۲ = -۳ & \text{ق ق} \end{cases}$

$\longrightarrow \tan x/۲ = -۳$

(۹

$\frac{\sin\theta}{۱-\cos\theta} + \frac{۱+\cos\theta}{\sin\theta} = \cot\frac{\alpha}{۲} + \cot\frac{\alpha}{۲} = ۲\cot\frac{\alpha}{۲} = k\cot\frac{\alpha}{۲} \longrightarrow k = ۲$

$\frac{۱-\cos\alpha}{\sin\alpha} = \frac{۲\sin^۲\frac{\alpha}{۲}}{۲\sin\frac{\alpha}{۲}\cdot\cos\frac{\alpha}{۲}} = \tan\frac{\alpha}{۲} \longrightarrow \frac{\sin\alpha}{۱-\cos\alpha} = \cot\frac{\alpha}{۲}$

$\frac{\sin\alpha}{۱+\cos\alpha} = \frac{۱-\cos\alpha}{\sin\alpha} = \tan\frac{\alpha}{۲} \longrightarrow \frac{۱+\cos\alpha}{\sin\alpha} = \cot\frac{\alpha}{۲}$

$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)=?$

$\sin\alpha=\frac{\sqrt{۲}}{۱۰} \rightarrow \cos^2\alpha=1-\sin^2\alpha=۱-\frac{۲}{۱۰۰}=\frac{۹۸}{۱۰۰} \rightarrow \cos\alpha=\sqrt{\frac{۹۸}{۱۰۰}}=\frac{۲\sqrt{۲۴}}{۱۰}$

$=\frac{۱}{۵}\sqrt{۲۴}$

$\cos\left(\frac{۱۱\pi}{۴}\right)=\cos\frac{۳\pi}{۴}=\cos\left(\pi-\frac{\pi}{۴}\right)=-\cos\frac{\pi}{۴}=-\frac{\sqrt{۲}}{۲}$

$\sin\left(\frac{۱۱\pi}{۴}\right)=\sin\frac{\pi}{۴}=\frac{\sqrt{۲}}{۲}$

$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)=\left(\cos\frac{۱۱\pi}{۴}\cdot\cos\alpha\right)-\left(\sin\frac{۱۱\pi}{۴}\cdot\sin\alpha\right)=$

$\left(-\frac{\sqrt{۲}}{۲}\times\frac{۲\sqrt{۲۴}}{۱۰}\right)-\left(\cancel{\frac{\sqrt{۲}}{۲}}\times\frac{\sqrt{۲}}{۱۰}\right)=\frac{-\cancel{۲}\sqrt{۲۴}}{\cancel{۲}۰}-\frac{۱}{۱۰}=\frac{-\sqrt{۲۴}-۱}{۱۰}$

$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)=-\left(\cos\alpha\cos\frac{\pi}{۴}+\sin\alpha\sin\frac{\pi}{۴}\right)$

$\rightarrow \frac{-\sqrt{۲}}{۲}(\cos\alpha+\sin\alpha)$ $\qquad \cos\alpha=\frac{-۷\sqrt{۲}}{۱۰}$

$\hookrightarrow \frac{-\sqrt{۲}}{۲}\left(\frac{-۷\sqrt{۲}}{۱۰}+\frac{\sqrt{۲}}{۱۰}\right)=\frac{۳}{۵}$

۳) $\dfrac{\sin^2 x+\cos^2 x}{\sin x\cos x} = -۳ \longrightarrow \sin x\cos x = \dfrac{-1}{۳} = A$

$\dfrac{1}{\sin^3 x+\cos^3 x} = \dfrac{1}{(\sin x+\cos x)(1-\sin x\cos x)}$

$A^2 = \sin^2+\cos^2 x+2\sin x\cos x = \dfrac{1}{۳}$

$\rightarrow A \begin{cases} \dfrac{1}{\sqrt{۳}} \times \\ -\dfrac{1}{\sqrt{۳}} \checkmark \end{cases}$

$\rightarrow \dfrac{-9}{4\sqrt{۳}} = -۰{,}۷۵\sqrt{۳}$

۴) $\cos\theta = \dfrac{x}{5} = \dfrac{6}{10}$ ۳

۴ $\sin\theta = \dfrac{h}{5} = \dfrac{8}{10}$

$S = \dfrac{(2+8)}{2} \times 4 = 20$

۶) $A = \sqrt{۳} \times \dfrac{-\sqrt{۳}}{2} \times \sin(270-27) - \sqrt{2} \times \dfrac{\sqrt{2}}{2}\cos(180-27)$

$\rightarrow \dfrac{5}{2}\cos(27) \longrightarrow$ برابر $\dfrac{5}{2}$

۷) $f\left(\dfrac{\pi}{36}\right) = 16\cos^2\left(\dfrac{\pi}{12}\right)\cos^2\left(\dfrac{\pi}{6}\right)\cos^2\left(\dfrac{\pi}{3}\right)\cos^2\left(\dfrac{2\pi}{3}\right)$

$\cos^2\dfrac{\pi}{12} = \dfrac{1+\cos\frac{\pi}{6}}{2} = \dfrac{2+\sqrt{3}}{4}$

$16\left(\dfrac{2+\sqrt{3}}{4}\right) \times \dfrac{3}{4} \times \dfrac{1}{4} \times \dfrac{1}{4}$

$= \dfrac{3(2+\sqrt{3})}{16}$