دیانا بابایی — یازدهم دفتر ۹ — تکلیف شماره ۲۸

۱)

$$\frac{1}{\sqrt{\cos^2\alpha}}-\frac{1}{\cot\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|}\xrightarrow{(\star)}\frac{1}{|\cos\alpha|}-\frac{|\sin\alpha|}{\cos\alpha}=\frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\cot\alpha=\frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}}\ (\star)\qquad \sqrt{\sin^2\alpha}=|\sin\alpha|$$

چون $|\sin\alpha|=\sin\alpha \leftarrow \sin\alpha>0$

چون $|\cos\alpha|=\cos\alpha \leftarrow \cos\alpha>0$ ← α در ربع اول

۲)

$$-\frac{\pi}{12}<x<\frac{5\pi}{12}\ ,\ \sin 2x=\frac{m-1}{4}\qquad m=?$$

$$-\frac{\pi}{6}<2x<\frac{5\pi}{6}\Rightarrow \sin(-\frac{\pi}{6})<\sin 2x<\sin(\pi-\frac{\pi}{6})\to -\frac{1}{2}<\sin 2x<\frac{1}{2}\Rightarrow$$

$$-\frac{1}{2}<\frac{m-1}{4}<\frac{1}{2}\to \begin{cases}(\frac{m-1}{4})<(\frac{1}{2})\xrightarrow{\times 4} m-1<2\to m<3\\ (\frac{m-1}{4})>(-\frac{1}{2})\xrightarrow{\times 4} m-1>-2\to m>-1\end{cases}$$

$$-1<m<3$$

۳)

$$\tan\alpha+\cot\alpha=-3,\quad 3\pi<4x<4\pi\to\frac{3\pi}{4}<x<\pi\to$$

$$\sin\frac{\pi}{4}<\sin x<\sin\pi\to \frac{\sqrt2}{2}<\sin x<0$$

$$-\cos\frac{\pi}{4}<\cos x<\cos\pi,\quad -\frac{\sqrt2}{2}<\cos x<-1\to -1<\sin x+\cos x<0\ \star$$

$$\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}=\frac{\sin^2\alpha+\cos^2\alpha}{\sin\alpha\cdot\cos\alpha}=\frac{1}{\sin\alpha\cdot\cos\alpha}=-3\to \sin\alpha\cdot\cos\alpha=\frac{-1}{3}\ (\star\star)$$

$$\sin\alpha=\alpha,\ \cos\alpha=\beta\to\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\to$$

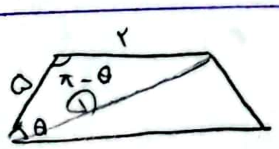
$$\sin^3\alpha+\cos^3\alpha=(\sin x+\cos x)^3-3\sin x\cdot\cos x(\sin x+\cos x)\ \frac{(\star),(\star\star)}{(\star\star\star)}$$

$$(-\frac{1}{\sqrt3})^3-3(-\frac{1}{3})(-\frac{1}{\sqrt3})=(\frac{-1}{\sqrt3})^3-\frac{1}{\sqrt3}=\frac{-2}{3\sqrt3}$$

$$(\sin\alpha+\cos\alpha)^2=\sin^2\alpha+\cos^2\alpha+2\underbrace{\sin\alpha\cdot\cos\alpha}_{-1/3}\to(\sin\alpha+\cos\alpha)^2=\frac{1}{3}\xrightarrow{\sin\alpha+\cos\alpha<0}$$

$$\sin\alpha+\cos\alpha=\frac{-1}{\sqrt3}\ (\star\star\star)$$

۴)



$$\cos\theta=0.6\ ,\ \cos(\pi-\theta)=-\cos\theta=-0.6$$

$$\sin(\underbrace{\pi-\theta}_{\alpha})\Rightarrow\sin^2\alpha=1-\cos^2\alpha=1-\frac{9}{25}=\frac{16}{25}\to\sin\alpha=\frac{4}{5}$$

$$S_{①}=\frac{1}{2}\times2\times5\times\frac{4}{5}=4\longrightarrow S=2\times4=8$$

$\tan(285^\circ)\cdot\tan(-165)-\sin(1095)\cos(255)$ (5

$\tan(\cancel{360}-75)\times\tan(-(180-15))-\sin(4\times\cancel{360}+15)\cdot\cos(180+75)=$

$-\tan(180-15)$ $\quad +90$

$\underbrace{-\tan 75\times\tan 15}_{-1}-\underbrace{\sin(15)\times-\cos(75)}_{\sin^2 15}=-1+\sin^2 15$

$\cancel{-1+\sin^2 15}=k\cos^2 15\longrightarrow k=1$

$\cancel{\cos^2 15}$

$A=\sqrt{3}\cos(210)\sin(243)-\sqrt{2}\sin(135)\cos 153$ (6

$A=\sqrt{3}\underbrace{\cancel{\cos(180+30)}}^{-\cos 30}\cdot\sin 63-\sqrt{2}\overbrace{\sin(\cancel{180-45})}^{\sin 45}\cdot\cos(180-27)$

$-\sqrt{3}\times\frac{1}{2}\times\sin 63-\sqrt{2}\times\frac{\sqrt{2}}{2}\times-\cos 27=-\frac{\sqrt{3}}{2}\underbrace{\cancel{\sin 63}}_{\cos 27}+\cos 27=$

$\cos 27\left(-\frac{\sqrt{3}}{2}+1\right)=\left(\frac{2-\sqrt{3}}{2}\right)\cos 27$

$f(x)=16\cos^2(3x)\cos^2(6x)\cos^2(12x)\cos^2(24x)\qquad\frac{\pi}{36}=5$ (7

$f\left(\frac{\pi}{36}\right)=f(5)=16\cos^2(15)+\underbrace{\cancel{\cos^2(90)}}_{0}\cos^2(180)\cos(120)=0$

$\frac{1-\sin x}{1+\sin x}=4\qquad\tan\frac{x}{2}=?$ (8

$4+4\sin x=1-\sin x\to 5\sin x=-3\to\sin x=\frac{-3}{5},\ \cos^2 x=\frac{16}{25}\to$

$\left.\cos x=-\frac{4}{5}\right\}\quad\tan x=\frac{3}{4}\qquad\frac{x}{2}=\alpha\to x=2\alpha\Rightarrow$

$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}\to\tan x=\frac{2\tan x/2}{1-\tan^2 x/2}\Rightarrow\frac{3}{4}=\frac{2\tan x/2}{1-\tan^2 x/2}\longrightarrow$

$3-3\tan^2 x/2=8\tan x/2\longrightarrow 3\tan^2 x/2+8\tan x/2-3=0\longrightarrow$

$\tan x/2=t\to 3t^2+8t-3=0\to\Delta=100\to\begin{cases}t_1=\frac{-8+10}{6}=\frac{2}{6}=\frac{1}{3} & \text{غ ق ق}\\ t_2=-3 & \text{ق ق}\end{cases}$

$\to\tan x/2=3$

$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\cot\frac{\alpha}{2}+\cot\frac{\alpha}{2}=2\cot\frac{\alpha}{2}=k\cot\frac{\alpha}{2}\to k=2$ (9

$\frac{1-\cos\alpha}{\sin\alpha}=\frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cdot\cos\frac{\alpha}{2}}=\tan\frac{\alpha}{2}\longrightarrow\frac{\sin\alpha}{1-\cos\alpha}=\cot\frac{\alpha}{2}$

$\frac{\sin\alpha}{1+\cos\alpha}=\frac{1-\cos\alpha}{\sin\alpha}=\tan\frac{\alpha}{2}\longrightarrow\frac{1+\cos\alpha}{\sin\alpha}=\cot\frac{\alpha}{2}$

$$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)=?$$

$$\sin\alpha=\frac{\sqrt{۲}}{۱۰}\rightarrow\cos^{۲}\alpha=۱-\sin^{۲}\alpha=۱-\frac{۲}{۱۰۰}=\frac{۹۸}{۱۰۰}\rightarrow\cos\alpha=\sqrt{\frac{۹۸}{۱۰۰}}=\frac{۲\sqrt{۲۴}}{۱۰}$$

$$=\frac{۱}{۵}\sqrt{۲۴}$$

$$\cos\left(\frac{۱۱\pi}{۴}\right)=\cos\frac{۳\pi}{۴}=\cos\left(\pi-\frac{\pi}{۴}\right)=-\cos\frac{\pi}{۴}=-\frac{\sqrt{۲}}{۲}$$

$$\sin\left(\frac{۱۱\pi}{۴}\right)=\sin\frac{\pi}{۴}=\frac{\sqrt{۲}}{۲}$$

$$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)=\left(\cos\frac{۱۱\pi}{۴}\cdot\cos\alpha\right)-\left(\sin\frac{۱۱\pi}{۴}\cdot\sin\alpha\right)=$$

$$\left(-\frac{\sqrt{۲}}{۲}\times\frac{۲\sqrt{۲۴}}{۱۰}\right)-\left(\frac{\sqrt{۲}}{\cancel{۲}}\times\frac{\sqrt{۲}}{۱۰}\right)=-\frac{\overset{۱}{\cancel{۲}}\sqrt{۲۴}}{\cancel{۲}۰}-\frac{۱}{۱۰}=\frac{-\sqrt{۲۴}-۱}{۱۰}$$