۱۹,۷۵

پرسش‌های چندگزینه‌ای

۱-

$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow$ ($\frac{1}{\sqrt{\cos^2\alpha}} = \frac{1}{|\cos\alpha|}$)

$\frac{-1}{\cot\alpha} = \frac{1-\sin\alpha-1}{|\cos\alpha|} \rightarrow$ ($\frac{-1}{\cot\alpha} = -\tan\alpha$)

$\frac{-\sin\alpha}{\cos\alpha} = \frac{-\sin\alpha}{|\cos\alpha|} \rightarrow \cos\alpha > 0$

$\cot\alpha = \frac{\cos\alpha}{\sqrt{\frac{1-\cos^2\alpha}{\sin^2\alpha}}} \rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \rightarrow \sin\alpha > 0$

ربع اول

۲-

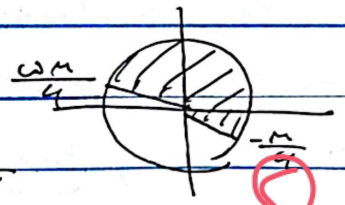
$\frac{-\pi}{12} < \theta < \frac{3\pi}{12}$

$\times 2 \rightarrow -\frac{\pi}{6} < 2\theta < \frac{3\pi}{6}$

$\max \sin 2\theta = 1 \leftarrow \sin\frac{\pi}{2}$

$\min \sin 2\theta = \frac{-1}{2} \leftarrow \sin\frac{-\pi}{6}$

$\sin 2\theta = \frac{m-1}{4}$



$\frac{-1}{2} < \underbrace{\sin 2\theta}_{\frac{m-1}{4}} \leq 1$

$\frac{m-1}{4} \leq 1 \rightarrow m-1 \leq 4 \rightarrow m \leq 5$

$\frac{m-1}{4} > \frac{-1}{2} \rightarrow \frac{m-1}{4} > \frac{-2}{4} \rightarrow m-1 > -2 \rightarrow m > -1$

$m \in (-1, 5]$

۳- دو راه حل

$$\tan\alpha+\cot\alpha=\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}\xrightarrow{\text{مخرج مشترک}}\frac{\overbrace{\sin^2\alpha+\cos^2\alpha}^{1}}{\sin\alpha\cos\alpha}=-3$$

$$\sin\alpha\cos\alpha=\frac{-1}{3}$$

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$$\frac{1}{\sin^3\alpha+\cos^3\alpha}=\frac{1}{(\sin\alpha+\cos\alpha)(\sin^2\alpha-\sin\alpha\cos\alpha+\cos^2\alpha)}=\frac{1}{\frac{-4}{3\sqrt{3}}}=\frac{-3\sqrt{3}}{4}$$

$\sin\alpha+\cos\alpha=\frac{-1}{\sqrt{3}}$

$\sin^2\alpha-\sin\alpha\cos\alpha+\cos^2\alpha=1+\frac{1}{3}=\frac{4}{3}$

$\sin\alpha\cos\alpha=\frac{-1}{3}$

$\sin^2\alpha+\cos^2\alpha=1$

$$(\sin\alpha+\cos\alpha)^2=\underbrace{\sin^2\alpha+\cos^2\alpha}_{1}+\underbrace{2\sin\alpha\cos\alpha}_{\frac{-2}{3}}=\frac{1}{3}\Rightarrow\sin\alpha+\cos\alpha=\pm\frac{1}{\sqrt{3}}$$

مثبت غ ق

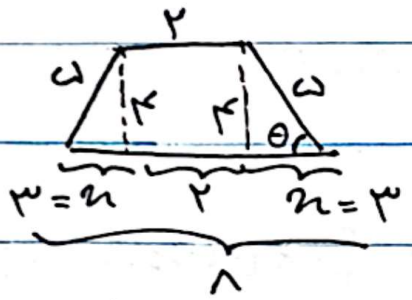
$3\pi<\alpha<4\pi\rightarrow\frac{3\pi}{4}<\alpha<\pi\rightarrow\sin\alpha>0$

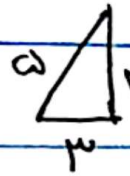
$\cos\alpha<0$

$\rightarrow\sin\alpha<\cos\alpha$

۴- $\cos\theta = 0/6 \rightarrow \frac{x}{5} = 0/6 \rightarrow x = 3$

$S_{\triangle} = \frac{\text{قاعده کوچک} + \text{قاعده بزرگ}}{2} \times \text{ارتفاع} = \frac{2+5}{2} \times 4 = 20$





مثلث طلایی

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پریا احسنی

ب- برابر چند است؟

$$\tan(\text{۲۸}\alpha)\tan(-\text{۱۹}\alpha)-\sin(\text{۱۰۹}\alpha)\cos(\text{۲۵}\alpha)=$$

$$\tan(\text{۲۷۰}+\text{۱}\alpha)\tan(-\text{۱۸۰}+\text{۱}\alpha)-\sin(\text{۱۰۸۰}+\text{۱}\alpha)\cos(\text{۲۷۰}-\text{۱}\alpha)$$

$\rightarrow \text{۳}\times\text{۳۶۰}$

$$\rightarrow(-\cot\text{۱}\alpha)\ \underbrace{\tan(-(\text{۱۸۰}-\text{۱}\alpha))}_{-\tan(\text{۱۸۰}-\text{۱}\alpha)=\tan\text{۱}\alpha}\ -(\sin\text{۱}\alpha)(-\sin\text{۱}\alpha)$$

$$\underbrace{(-\cot\text{۱}\alpha)(\tan\text{۱}\omega)}_{-\text{۱}}+\sin^{\text{۲}}\text{۱}\alpha=\sin^{\text{۲}}\text{۱}\alpha-\text{۱}=-(\text{۱}-\sin^{\text{۲}}\text{۱}\alpha)=-\cos^{\text{۲}}\text{۱}\alpha$$

$$=\text{۱}\kappa\cos^{\text{۲}}\text{۱}\alpha\longrightarrow\text{۱}\kappa=\boxed{-\text{۱}}$$

ج- برابر چند است؟

$$\sqrt{\text{۳}}\cos(\text{۲۱۰})\sin(\text{۲۴۳})-\sqrt{\text{۲}}\sin(\text{۱۳۵})\cos(\text{۱۵۳})=$$

$\text{۲۱۰}=\text{۱۸۰}+\text{۳۰}$ ، $\text{۲۴۳}=\text{۲۷۰}-\text{۲۷}$ ، $\text{۱۳۵}=\text{۱۸۰}-\text{۴۵}$ ، $\text{۱۵۳}=\text{۱۸۰}-\text{۲۷}$

$$(-\sqrt{\text{۳}}\cos\text{۳۰})(-\cos\text{۲۷})-(\sqrt{\text{۲}}\sin\text{۴۵})(-\cos\text{۲۷})=\frac{-\text{۳}}{\text{۲}}\times(-\cos\text{۲۷})-(-\cos\text{۲۷})$$

$-\sqrt{\text{۳}}\times\frac{\sqrt{\text{۳}}}{\text{۲}}=\frac{-\text{۳}}{\text{۲}}$ ، $\sqrt{\text{۲}}\times\frac{\sqrt{\text{۲}}}{\text{۲}}=\text{۱}$

$$=\frac{\text{۳}}{\text{۲}}\cos\text{۲۷}+\cos\text{۲۷}=\frac{\text{۵}}{\text{۲}}\cos\text{۲۷}$$

$\boxed{\frac{\text{۵}}{\text{۲}}\text{ برابر}}$

$$\frac{\frac{\text{۵}}{\text{۲}}\cancel{\cos\text{۲۷}}}{\cancel{\cos\text{۲۷}}}=\frac{\text{۵}}{\text{۲}}$$

۷-

$$14\cos^2\left(\frac{3\pi}{24}\right)\cos^2\left(\frac{4\pi}{24}\right)\cos^2\left(\frac{12\pi}{24}\right)\cos^2\left(\frac{24\pi}{24}\right)$$

$\frac{\pi}{12}$ $\frac{\pi}{6}$ $\frac{\pi}{3}$ $\frac{2\pi}{3}$

$$\cos^2\frac{\pi}{12}=\frac{1+\cos\frac{\pi}{6}}{2}=\frac{\frac{2+\sqrt{3}}{2}}{2}=\frac{2+\sqrt{3}}{4} \quad / \quad \cos^2\frac{\pi}{6}=\frac{1+\cos\frac{\pi}{3}}{2}=\frac{\frac{3}{2}}{2}=\frac{3}{4} \quad /$$

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2}$$

$$\cos^2\frac{\pi}{3}=\frac{1+\cos\frac{2\pi}{3}}{2}=\frac{\frac{1}{2}}{2}=\frac{1}{4} \quad /$$

$$\cos\frac{2\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}=-\frac{1}{2}$$

$$\cos\frac{4\pi}{3}=\cos\left(\pi+\frac{\pi}{3}\right)=-\cos\frac{\pi}{3}=-\frac{1}{2}$$

$$\cos^2\frac{2\pi}{3}=\frac{1+\cos\frac{4\pi}{3}}{2}=\frac{\frac{1}{2}}{2}=\frac{1}{4}$$

$$14\left(\frac{2+\sqrt{3}}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)=\frac{9+3\sqrt{3}}{14}$$

۸-

$$\frac{1-\sin\theta}{1+\sin\theta}=4 \longrightarrow 4+4\sin\theta=1-\sin\theta \longrightarrow 5\sin\theta=-3 \longrightarrow \sin\theta=\frac{-3}{5}$$

$$\sin^2\theta+\cos^2\theta=1 \longrightarrow \frac{9}{25}+\cos^2\theta=\frac{25}{25} \longrightarrow \cos^2\theta=\frac{16}{25} \xrightarrow{\text{ربع ۳}} \cos\theta=-\frac{4}{5}$$

$$\tan\frac{\theta}{2}\xrightarrow{\frac{\theta}{2}=\alpha}\tan\alpha \quad / \quad \tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha} \longrightarrow \tan\theta=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{-3}{5}}{\frac{-4}{5}}=\frac{3}{4}$$

$$\frac{3}{4}=\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} \longrightarrow 3-3\tan^2\frac{\theta}{2}=8\tan\frac{\theta}{2} \longrightarrow 3\tan^2\frac{\theta}{2}+8\tan\frac{\theta}{2}-3=0$$

$$ac \longrightarrow \tan^2\frac{\theta}{2}+8\tan\frac{\theta}{2}-9=0 \xrightarrow[\text{منفی شده}]{\text{جمع ضرایب}} \tan\frac{\theta}{2}=1 \xrightarrow{\div 3} \frac{1}{3}$$

$$\tan\frac{\theta}{2}=-9 \xrightarrow{\div 3} -3$$

غ ق ق

$\tan>0$ ربع ۳

$\frac{\theta}{2}$ ربع ۳ می باشد پس $\frac{\theta}{2}$ در ربع ۲ قرار دارد بنابراین $\tan\frac{\theta}{2}<0$

۹- برابر است با

$$\frac{\sin\theta}{1-\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=\frac{1}{\tan\frac{\theta}{2}}+\frac{1}{\tan\frac{\theta}{2}}=\cot\frac{\theta}{2}+\cot\frac{\theta}{2}=$$

$$2\cot\frac{\theta}{2}=k\cot\frac{\theta}{2}$$

$$k=2$$

۹

$$\frac{1-\cos\alpha}{\sin\alpha}=\tan\frac{\alpha}{2}$$

$$\frac{\sin\alpha}{1+\cos\alpha}=\tan\frac{\alpha}{2}$$

فرعی

۱۰- پرسپولیس

$\sin\alpha > 0 \quad \cos\alpha < 0$

$\sin\alpha = \frac{\sqrt{۲}}{۱۰}$ $\qquad \sin^۲\alpha + \cos^۲\alpha = ۱ \rightarrow (\frac{\sqrt{۲}}{۱۰})^۲ + \cos^۲\alpha = ۱ \rightarrow \cos^۲\alpha = \frac{۴۹}{۵۰}$ $\quad \frac{۲}{۱۰۰}$

$\cos\alpha = -\sqrt{\frac{۴۹}{۵۰}} = -\frac{۷}{\sqrt{۵۰}} = -\frac{۷}{۵\sqrt{۲}} = -\frac{۷\sqrt{۲}}{۱۰}$

$\frac{۱۱\pi}{۴} = ۲\pi + \frac{\pi}{۴}$

$\cos(\frac{۱۱\pi}{۴} + \alpha) = \cos(\frac{\pi}{۴} + \alpha)$

۵

فرمول : $\cos(a+b) = \cos a\cos b - \sin a\sin b$

$\cos(\frac{\pi}{۴} + \alpha) = \underbrace{\cos\frac{\pi}{۴}}\cos\alpha - \underbrace{\sin\frac{\pi}{۴}}\sin\alpha =$

$\cos(\pi - \frac{\pi}{۴}) = -\cos\frac{\pi}{۴}$ $\qquad \sin(\pi - \frac{\pi}{۴}) = \sin\frac{\pi}{۴}$

$(-\frac{\sqrt{۲}}{۲} \times \frac{-۷\sqrt{۲}}{۱۰}) - (\frac{\sqrt{۲}}{۲} \times \frac{\sqrt{۲}}{۱۰}) = \frac{۷}{۱۰} - \frac{۱}{۱۰} = \frac{۶}{۱۰} = \boxed{\frac{۳}{۵}}$