

Arithmetik

Algebra

10. Klasse

$$\textcircled{1} \lim_{x \rightarrow 1} \frac{x^p - \sqrt[p]{x+p}}{a x^p - 1 x + p} = \frac{x(x-1)(x-\frac{p}{a})}{a(x-1)(x-\frac{p}{a})} =$$

$$\lim_{x \rightarrow 1} \frac{x-p}{a x - p} = \boxed{\frac{1}{a}}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{|x-1| - |x+1|}{x} \Rightarrow \lim_{x \rightarrow 0^+} \frac{|x-1| - |x+1|}{x} = \frac{-x+1 - x-1}{x} = \frac{-2x}{x} = -2$$

$$\hookrightarrow = -2$$

$$\lim_{x \rightarrow 0^-} \frac{|x-1| - |x+1|}{x} = \frac{-x+1 - x-1}{x} = \frac{-2x}{x} = -2$$

$$\textcircled{3} \lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x}-2} = \frac{(x+2)(\sqrt{x}-2)}{\sqrt{x}-2} = (\sqrt{x}+2) = \boxed{6}$$

$$\textcircled{4} \lim_{x \rightarrow 4} \frac{x - \sqrt{4x}}{x^2 - x - 4} = \frac{x - \sqrt{4x}}{(x-2)(x+2)} \times \frac{x + \sqrt{4x}}{x + \sqrt{4x}}$$
$$= \frac{x^2 - 4x}{(x-2)(x+2)(x + \sqrt{4x})} = \frac{x(x-4)}{(x-2)(x+2)(x + \sqrt{4x})} = \frac{4}{\sqrt{4x}} = \boxed{\frac{1}{\sqrt{4}}}$$



$$\textcircled{a} \lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{x - \sqrt{x-2}} = x \frac{1 + \sqrt{x}}{1 + \sqrt{x}} \times \frac{x + \sqrt{x-2}}{x + \sqrt{x-2}}$$

$$= \frac{1 - x}{x - \sqrt{x-2}} \times \frac{x + \sqrt{x-2}}{1 + \sqrt{x}} = -1 \times \frac{x}{x} = \boxed{-1}$$

$$\textcircled{b} \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - 2}{\sqrt{x+2} - 3} \times \frac{\sqrt{x+2} + 2}{\sqrt{x+2} + 2} \times \frac{\sqrt{(x+2)^2} + \sqrt{x+2} + 9}{\sqrt{(x+2)^2} + \sqrt{x+2} + 9}$$

$$= \frac{x+2 - 4}{x+2 - 6} \times \frac{x+2}{\sqrt{x+2} + 2} = \frac{x-2}{x-4} \times \frac{x+2}{\sqrt{x+2} + 2} = \frac{x-2}{x-4} \times \frac{x+2}{\sqrt{x+2} + 2} = \frac{11}{5}$$

$$\textcircled{c} \lim_{x \rightarrow 1} \frac{\sqrt{x+\sqrt{x}} - 2}{\sqrt{x} - 1} \times \frac{\sqrt{x+\sqrt{x}} + 2}{\sqrt{x+\sqrt{x}} + 2} \times \frac{\sqrt{x^2} + \sqrt{x} + 1}{\sqrt{x^2} + \sqrt{x} + 1}$$

$$= \frac{x+\sqrt{x} - 4}{x-1} \times \frac{x}{\sqrt{x+\sqrt{x}} + 2} \xrightarrow{\text{hop}} \frac{x + \frac{1}{\sqrt{x}}}{1} \times \frac{x}{x} = \frac{2}{1} \times \frac{1}{1} = \boxed{\frac{2}{1}}$$

$$\textcircled{d} \lim_{x \rightarrow \pi} \frac{1 + \cos^2 x}{\sin^2 x} = \frac{0}{0} = \frac{(1 + \cos x)(1 - \cos x + \cos^2 x)}{1 - \cos^2 x}$$

$$\frac{(1 + \cos x)(1 - \cos x + \cos^2 x)}{(1 + \cos x)(1 - \cos x)} = \frac{1 + 1 + 1}{1} = \boxed{3}$$

$$\textcircled{9} \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \tan x}{\sin x - \cos x} = \frac{1 - \frac{\sin x}{\cos x}}{\sin x - \cos x} = \frac{\cos x - \sin x}{\cos x (\sin x - \cos x)}$$

$$\frac{\cos x - \sin x}{(\cos x)(\sin x - \cos x)} = \frac{-1}{\cos x} = \frac{-1 \times \sqrt{x}}{\sqrt{x}} = \frac{-\sqrt{x}}{\sqrt{x}} = -\sqrt{x}$$

$$\textcircled{10} \lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan^2 x - 1}{\cos^2 x} = \frac{(\tan x - 1)(\tan x + 1)}{\cos^2 x - \sin^2 x} = \frac{\frac{\sin x - \cos x}{\cos x} \times \frac{\sin x + \cos x}{\cos x}}{(\cos x + \sin x)(\cos x - \sin x)}$$

$$\frac{(\sin x - \cos x)(\sin x + \cos x)}{\cos^2 x \times (\cos x + \sin x)(\cos x - \sin x)} = \frac{-1}{\cos^2 x} = \frac{-1}{\frac{\sqrt{x}}{\sqrt{x}}} = \frac{-\sqrt{x}}{\sqrt{x}} = -\sqrt{x}$$