آنالیز ریاضی — با استفاده از تشریح ۳۰

① $\lim_{x\to 1} \frac{6x^2-7x+1}{5x^2-4x+3} = \frac{0}{0} = \frac{(x-1)(x-\frac{1}{6})}{(x+2)(x-\frac{3}{5})} = \frac{1-\frac{1}{6}}{1-\frac{3}{5}} = \frac{\frac{5}{6}}{\frac{2}{5}} = \frac{5}{8}$

② $\lim_{x\to 0} \frac{|3x-1|-|3x+1|}{x} \times \frac{\text{مزدوج}}{\text{مزدوج}} = \frac{|9x^2-1|}{x(x+2)}$ $\begin{cases} 0^+ \to \frac{1}{0^+} = +\infty \\ 0^- \to \frac{1}{0^-} = -\infty \end{cases}$

③ $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2}$ ، $\sqrt{x}+2=4$

④ $\lim_{x\to 2} \frac{x-\sqrt{3x}}{2x^2-x-6} = \frac{0}{0} = \frac{x-\sqrt{3x}}{(x-2)(x+\frac{3}{2})} \times \frac{x+\sqrt{3x}}{x+\sqrt{3x}} = \frac{x^2-3x}{(x-2)(x+\frac{3}{2})(x+\sqrt{3x})} = \frac{x}{(x+\frac{3}{2})x} = \frac{2}{7} = \frac{1}{7}$

⑤ $\lim_{x\to 1} \frac{1-\sqrt{x}}{4-\sqrt{5-x}} \times \frac{4+\sqrt{5-x}}{4+\sqrt{5-x}} = \frac{(1-x)(4+\sqrt{5-x})}{(16-5+x)-x^2} = \frac{(1-\sqrt{x})(1+\sqrt{x})(4+\sqrt{5-x})}{(1-x)(1+x)} = \frac{4+4}{2} = -2$

⑥ $\lim_{x\to 4} \frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3} = \frac{0}{0} \times \frac{\text{مزدوج}}{\text{مزدوج}} \times \frac{\text{مکعب}}{\text{مکعب}} = \frac{(3x+4-16)(25)}{(5x+7-27)8} = \frac{(3x-12)(25)}{(5x-20)8} = \frac{(x-4)3\times 25}{(x-4)5\times 8} = \frac{15}{40}$

⑦ $\lim_{x\to 1} \frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1} = \frac{0}{0} \cdot \frac{\text{مزدوج}}{\text{مزدوج}} = \frac{(3x+\sqrt{x}-4)(x^2+x+1)}{(x-1)(x^2+2x+1)} = \frac{9x-3\sqrt{x}-12}{3x-3} = \frac{3(\sqrt{x}-1)(\sqrt{x}+\frac{4}{3})}{3(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{7}{8}$

⑧ $\lim_{x\to \frac{\pi}{4}} \frac{1-\tan x}{\sin x-\cos x} \times \frac{\sin x+\cos x}{\sin x+\cos x} = \frac{(1-\tan^2 x)(\sin x+\cos x)}{(\sin^2 x-\cos^2 x)(1+\tan x)} = \frac{(1-\tan^2 x)(\sin x+\cos x)}{(-\cos 2x)(1+\tan x)}$

⑨ $\lim_{x\to \pi} \frac{1+\cos^{2n+1} x}{\sin^2 x} = \frac{(1+\cos x)(1+\cos^2 x-\cos x)}{(1-\cos x)(1+\cos x)} = \frac{1+1-(-1)}{2} = \frac{3}{2}$

⑩ $\lim_{x\to \frac{\pi}{4}} \frac{\tan^2 x-1}{\cos^2 x} = \frac{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}{-(\sin^2 x-\cos^2 x)} = \frac{1}{-\cos^2 x} = \frac{1}{-(\frac{\sqrt{2}}{2})^2} = \frac{1}{-\frac{1}{2}} = -2$

⑪ $\lim_{x\to \frac{\pi}{4}} \frac{1+\tan x}{\sin x-\cos x} = \frac{\frac{\cos x-\sin x}{\cos x}}{-(\cos x-\sin x)} = \frac{1}{-\cos x} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$