نام و نام خانوادگی ....هستی گلدوزی جواه........ پاسخنامه تشریحی تکلیف شماره ۴.... کلاس ...یازدهم (A)........

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الف)

$\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \qquad x \neq 0,\ x-1 \neq 0,\ x \neq 1$ (1)

$\frac{x^2-2x+1-x^2}{x^2-x} \geq 0 \quad$ ریشه صورت $\frac{1}{2}$ $\quad \frac{-2x+1}{x^2-x} \geq 0$

ریشه مخرج $0, 1$

x		0		$\frac{1}{2}$		1	
$-2x+1$	+		+	0	−		−
x^2-x	+	0	−		−	0	+
	+		−	0	+		−

$(-\infty, 0) \cup [\frac{1}{2}, 1)$ (2)

$(1) \cap (2) \longrightarrow D_f: (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب)

$x \neq -1 \leftarrow x+1 \neq 0$

$x \neq 0 \leftarrow x \neq 0$

$x \neq 1 \leftarrow x-1 \neq 0$

$x \neq -2 \leftarrow x+2 \neq 0$

$\frac{3}{x-1} + \frac{1}{x+2} \neq 0$

$\frac{3}{x-1} \neq \frac{-1}{x+2}$

$3x+6 \neq -x+1$

$4x \neq -5$

$x \neq \frac{-5}{4}$

$D_f: \mathbb{R} - \{-2, -1, 1, 0, \frac{-5}{4}\}$

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الف)

$\left[(\frac{1}{3})^x - 9\right](32 - 4^x) \geq 0$

x		-2		$\frac{5}{2}$	
A	+	0	−		−
B	+		+	0	−
	+	0	−	0	+

ریشه: A: $(\frac{1}{3})^x - 9 = 0 \quad (\frac{1}{3})^x = 9 \quad x = \log_{\frac{1}{3}} 9 = -2$

B: $32 - 4^x = 0 \quad 4^x = 32 \quad x = \log_4 32 = \frac{5}{2}$

$D_f: (-\infty, -2] \cup [\frac{5}{2}, +\infty)$ منفی ۲

ب)

$\sqrt{y+1} = 3 - \sqrt[4]{x-1}$

$y+1 = 9 - 6\sqrt[4]{x-1} + \sqrt[2]{x-1}$

$y = 8 - 6\sqrt[4]{x-1} + \sqrt[2]{x-1}$

$x-1 \geq 0$

$x \geq 1$

$D_f: [1, +\infty)$

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$\sqrt{x^2-1} + 1 \neq 0$ همواره برقرار است

$\sqrt{x^2-1} \neq -1$

A: $x^2 - 1 \geq 0$

$x^2 \geq 1$

$x \geq 1$

$x \leq -1 \qquad (-\infty, -1] \cup [1, +\infty)$ (1)

$x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

ریشه ۲ ریشه ۱-

x		-1		2	
	+		−		+

$(-\infty, -1) \cup (2, +\infty)$ (2)

$(1) \cap (2) \longrightarrow (-\infty, -1) \cup (2, +\infty)$

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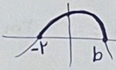
$\sqrt{3+ax-x^2} \rightsquigarrow -x^2+ax+3 \geq 0 \longrightarrow$



$D_{(f)} = [-2, b]$

$b = \frac{3}{2}$

$\boxed{\frac{3}{2} - \frac{1}{2} = 1 \quad b + 2a = 1}$

$x = -2 \longrightarrow f(x) = 0$

$-(-2)^2 + (-2)a + 3 = 0$

$-4 - 2a + 3 = 0$

$a = \frac{-1}{2}$

$$\begin{array}{l|l} -x^2 - \frac{1}{2}x + 3 & x+2 \\ \hline \cancel{-x^2} - 2x & -x + \frac{3}{2} \\ \frac{3}{2}x + 3 & \\ \frac{3}{2}x + 3 & \end{array}$$

یکی از ریشه‌ها = ۲- بنابراین عبارت بر $x+2$ بخش پذیر است.

ریشه دوم = $\frac{3}{2}$ در ریشه تحلیل برخورد

$-x^2 - \frac{1}{2}x + 3 = (x+2)(\frac{3}{2} - x)$

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$\sqrt{f(x) - x} \qquad f(x) - x \geq 0 \qquad \boxed{f(x) \geq x}$

$(x \geq 1)$: $3x - 2 \geq x \longrightarrow 2x \geq 2 \longrightarrow \boxed{x \geq 1}$ (1)

$(x < 1)$: $2x + 3 \geq x \longrightarrow x \geq -3 \rightsquigarrow [-3, +\infty) \cap (-\infty, 1) \xrightarrow{\text{اشتراک}} \boxed{[-3, 1)}$ (2)

$(1) \cup (2) \longrightarrow [1, +\infty) \cup [-3, 1) \Longrightarrow D(g): [-3, +\infty)$

$5>1 \longrightarrow f(5) = (a+1)(5+2) = 7(a+1) = 7a+7$

$-2<1 \longrightarrow f(-2) = 3a + 2(-2) = 3a-4$

$2f(5) = f(-2) + a$ ۶

$$\left.\begin{aligned} 2(7a+7) &= 3a-4+a \\ 2(7a+7) &= 4a-4 \\ \cancel{2}(7a+7) &= \cancel{2}(2a-2) \end{aligned}\right\} \quad \cancel{7a+7 = 4a-4} \quad 7a+7 = 2a-2 \quad 5a = -9 \quad a = \frac{-9}{5}$$

$2-\sqrt{3} = a \qquad f(a) = \sqrt{a} + \frac{1}{\sqrt{a}} + 2 \rightarrow f(a) = f(\tfrac{1}{a})$

$2+\sqrt{3} = \frac{1}{a} \qquad f(\tfrac{1}{a}) = \sqrt{\tfrac{1}{a}} + \frac{1}{\sqrt{\tfrac{1}{a}}} + 2 = \frac{1}{\sqrt{a}} + \sqrt{a} + 2 \qquad f(a) + f(\tfrac{1}{a}) = 2f(a)$

$2f(a) = 2\left(\sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2\right) = 2\left(\underbrace{\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}}_{} + 2\right) = 2(t+2)$ ۷

مجموع دو عبارت نشانی t همواره مثبت

$t^2 = 2-\cancel{\sqrt{3}} + 2 + \cancel{\sqrt{3}} + 2\sqrt{(2-\sqrt{3})(2+\sqrt{3})}$

$t^2 = 2+2+2\sqrt{1} \Rightarrow t^2 = 6 \Rightarrow \overset{\text{همواره مثبت}}{t = \sqrt{6}}$

$f(2-\sqrt{3}) + f(2+\sqrt{3}) = 2(t+2) = 2(\sqrt{6}+2) = \boxed{2\sqrt{6}+4}$

$$\begin{cases} 2f(x) - 3f(-x) = 4x^2 - x \\ \xrightarrow{-x} 2f(-x) - 3f(x) = 4(-x)^2 + x = 4x^2 + x \end{cases}$$

$\xrightarrow{\times 2} 4f(x) - 6f(\cancel{-x}) = 8x^2 - 2x$ ۸

$\xrightarrow{\times 3} -9f(x) + \cancel{6}f(-x) = 12x^2 + 3x$

$-5f(x) = 20x^2 + x$

$f(x) = \dfrac{20x^2 + x}{-5}$

$\xrightarrow{0} 2f(0) = 3m-1 \rightarrow f(0) = \dfrac{3m-1}{2}$

$\xrightarrow{-2} +6f(0) = 16 + 2m + 3m - 1 \rightarrow f(0) = \dfrac{15+5m}{6}$

$\dfrac{3m-1}{\cancel{2}_{1}} = \dfrac{15+5m}{{}_{3}\cancel{6}} \qquad f(0) = \dfrac{3(\frac{9}{2})-1}{2} = \dfrac{\frac{27}{2} - \frac{2}{2}}{2} = \boxed{\dfrac{25}{4}}$ ۹

$9m - 3 = 15 + 5m \rightarrow 4m = 18 \rightarrow m = \dfrac{9}{2}$

$f(x) + f(\tfrac{1}{x}) = ax + b + \frac{a}{x} + b = \dfrac{3x^2 - 12x + 3}{x}$

$\dfrac{ax^2 + bx + a + bx}{\cancel{x}} = \dfrac{3x^2 - 12x + 3}{\cancel{x}} \Longrightarrow \boxed{a = 3}$

$2b = -12 \rightarrow \boxed{b = -6}$ ۱۰

$\underline{\underline{f(-1)}} = ax + b = 3x - 6 = 3(-1) - 6 = -3 - 6 = \boxed{-9}$