ماهک میرمفتاحی

۱ ← الف

$$f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \rightarrow$$

$$\frac{\overbrace{(x-1)^2}^{x^2-2x+1}-x^2}{x(x-1)}=\frac{1-2x}{x(x-1)}\geq 0$$

$1-2x=0 \rightarrow \frac{1}{2}$, $\quad x=0$, $\quad x-1=0 \rightarrow 1$

	0		$\frac{1}{2}$		1	
$+$	$\circ$	$-$	$\bullet$	$+$	$\circ$	$-$

$$\boxed{D_f=(-\infty,0)\cup[\tfrac{1}{2},1)}$$

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۱ ← ج

$$f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$$

$$\frac{1}{x+1}-\frac{2}{x} \rightarrow \frac{x-2x-1}{x(x+1)}=\frac{-(x+1)}{x(x+1)}$$

$x=0$, $\quad x+1=0 \rightarrow -1$

$$\frac{3}{x-1}+\frac{1}{x+2} \rightarrow \frac{3x+6+x-1}{(x-1)(x+2)}=\frac{4x+5}{(x-1)(x+2)}$$

$x-1=0 \rightarrow 1$, $\quad x+2=0 \rightarrow -2$

$$\frac{4x+5}{(x-1)(x+2)}=0 \rightarrow 4x+5=0 \rightarrow x=-\frac{5}{4}$$

$$\boxed{D_f=\mathbb{R}-\{0,-1,1,-2,-\tfrac{5}{4}\}}$$

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۲ ← الف

$$f(x)=\sqrt{\left(\left(\tfrac{1}{3}\right)^x-9\right)\left(32-4^x\right)} \rightarrow$$

$$\left(\left(\tfrac{1}{3}\right)^x-9\right)\left(32-4^x\right)\geq 0$$

$\left(\tfrac{1}{3}\right)^x-9=0 \rightarrow -2$, $\quad 32-4^x=0 \rightarrow \frac{5}{2}$

	-2		$\frac{5}{2}$	
$+$	$\bullet$	$-$	$\bullet$	$+$

$\rightarrow$

$$\boxed{D_f = (-\infty, -2] \cup [5/2, \infty)}$$

۲ ← [illegible] ←

$$\sqrt[4]{x-1} + \sqrt{y+1} = 3 \Rightarrow \sqrt{y+1} = 3 - \sqrt[4]{x-1}$$

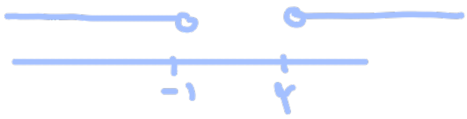
$$y = 9 - 1 + \sqrt{x-1} - 6\sqrt[4]{x-1} \Rightarrow y = 8 - 6\sqrt[4]{x-1} + \sqrt{x-1}$$

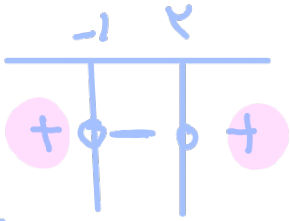
$$\left.\begin{array}{l} \rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \\ 3 - \sqrt[4]{x-1} \geq 0 \Rightarrow x-1 \leq 81 \Rightarrow x \leq 82 \end{array}\right\} \Rightarrow \boxed{D_f = [1, 82]}$$

۳ ←

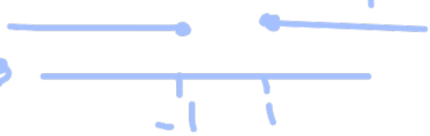
$$f(x) = \frac{\log_2 (x^2 - x - 2)}{\sqrt{x^2-1} + 1}$$

$$① \Rightarrow x^2 - x - 2 > 0 \rightarrow (\overset{2}{x-2})(\overset{-1}{x+1}) > 0$$





$$② \Rightarrow x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow$$



$$③ \Rightarrow \sqrt{x^2-1} + 1 \neq 0 \Rightarrow \underbrace{\sqrt{x^2-1}}_{\text{همیشه} +} \neq -1 \quad \times \quad \mathbb{R}$$

$$① \cap ② \cap ③ \Rightarrow \boxed{D_f = (-\infty, -1) \cup (2, +\infty)}$$

$$\sqrt{3 + 6x - x^2} \Rightarrow 3 + 6x - x^2 \geq 0$$

۴ ←

$$x^2 - ax - 3 \leq 0$$

$$-2 \times b = 3 \Rightarrow b = \frac{3}{2} \Rightarrow a = -\frac{4}{2} + \frac{3}{2} = -\frac{1}{2} \Rightarrow$$

$$\Rightarrow a + b = \frac{3}{2} + \frac{1}{2} = \frac{4}{2} = \boxed{1}$$

$$f(x) = \begin{cases} 3x - 2, & x \geq 1 \\ 2x + 3, & x < 1 \end{cases}$$ ← 5

$$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) - x \geq 0 \Rightarrow$$

$$f(x) \geq x \longrightarrow \left.\begin{array}{l} x \geq 1 \\ 2x + 3 \geq x \rightarrow x \geq -3 \end{array}\right\} \Rightarrow \boxed{D_f = [-3, +\infty)}$$

$$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a + 2x & x \leq 1 \end{cases}$$ ← 6

$$\left.\begin{array}{l} 2f(5) = f(-2) + a \\ \quad 7(a+1) \qquad 3a - 4 \end{array}\right\} \Rightarrow \begin{array}{l} 14a + 14 = 4a - 4 \\ 10a = -18 \rightarrow \boxed{a = -\frac{9}{5}} \end{array}$$

$$\frac{1}{۲+\sqrt{۳}} = ۲-\sqrt{۳} \Rightarrow f(\underbrace{۲+\sqrt{۳}}_{t}) + f\left(\frac{1}{۲+\sqrt{۳}}\right) \leftarrow ۷$$

$$f(t) + f\left(\frac{1}{t}\right) = ۲\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right) + ۴ \Rightarrow$$

$$۲\left(\sqrt{۲+\sqrt{۳}} + \frac{1}{\sqrt{۲+\sqrt{۳}}}\right) + ۴ \rightarrow$$

$$۲\left(\underbrace{\sqrt{۲+\sqrt{۳}} + \sqrt{۲-\sqrt{۳}}}_{S}\right) + ۴$$

ادامه سوال بعد از تمام سوالات

$$\left(۲f(x) - ۳f(-x) = ۴x^2 - x\right) \times ۳ \qquad \leftarrow ۸$$

$$\left(۲f(-x) - ۳f(x) = ۴x^2 + x\right) \times ۲$$

$$-۵f(x) = ۲۰x^2 + x = x(۲۰x+1) \Rightarrow$$

$$f(x) = \frac{-x(۲۰x+1)}{۵}$$

$$(x+۲)f(x) - ۳xf(x+۲) = ۴x^2 - mx + ۳m - 1 \qquad \leftarrow ۹$$

$x=0 \Rightarrow (2f(0) = 3m-1)\times -3 \quad 5m$

$x=-2 \Rightarrow 6f(0) = 15 + \cancel{2m} + 5m - 1$

$0 = 18 - 4m \Rightarrow 18 = 4m \Rightarrow m = \frac{9}{2}$

$2f(0) = \cancel{\frac{27}{2}} - \frac{2}{2} \quad \frac{25}{2} \Rightarrow \boxed{f(0) = \frac{25}{4} = 6{,}25}$

$f(x) + f(\frac{1}{x}) = ax + b + \frac{a}{x} + b \Rightarrow$ ← 10

$a(x + \frac{1}{x}) + 2b = 3x - 12 + \frac{3}{x} \Rightarrow$

$\left.\begin{matrix} a = 3 \\ b = -6 \end{matrix}\right\} \Rightarrow f(-1) = -1\times 3 - 6 \boxed{= -9}$

ادامه سوال ۷ ←

$S^2 = (2-\sqrt{3})(2+\sqrt{3}) + 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})}$

$\Rightarrow S^2 = 6$

$f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2S + 4 \Rightarrow$ $\quad S \to \sqrt{6}$

$$\boxed{f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2\sqrt{6} + 4}$$