

الف) $f(n) = \sqrt{\frac{n-1}{n} - \frac{n}{n-1}}$

$$\frac{(n-1)^2 - n^2}{n(n-1)} \geq 0 \rightarrow \frac{(n-1-n)(n-1+n)}{n(n-1)} \geq 0$$

$$\frac{-1}{n} + \frac{1}{n-1} \geq 0 \rightarrow \frac{1}{n-1} - \frac{1}{n} \geq 0 \rightarrow \frac{n - (n-1)}{n(n-1)} \geq 0 \rightarrow \frac{1}{n(n-1)} \geq 0$$

$$D_f = (-\infty, 0) \cup \left[\frac{1}{4}, 1\right)$$

ب) $f(n) = \frac{1}{n+1} - \frac{r}{n}$

$$\frac{1}{n+1} + \frac{1}{n+r} \geq 0 \rightarrow \frac{n+r+n+1}{(n+1)(n+r)} \geq 0 \rightarrow 2n+r+1 \geq 0 \rightarrow n \geq -\frac{r+1}{2}$$

$$D_f = \mathbb{R} - \left\{-r, -\frac{r}{2}, -1, 0, 1\right\}$$

الف) $f(n) = \sqrt{\left(\frac{1}{n}\right)^n - 9}(n-r-1)$

$$\left(\frac{1}{n}\right)^n - 9 \geq 0 \rightarrow \frac{1}{n^n} \geq 9 \rightarrow n^n \leq \frac{1}{9} \rightarrow n \leq -\frac{1}{3}$$

$$I) \frac{1}{n^n} - 9 = 0 \rightarrow n^n = 9 \rightarrow n = 3$$

$$II) n-r-1 = 0 \rightarrow n = r+1$$

$$D_f = (-\infty, -r] \cup \left[\frac{r}{3}, +\infty\right)$$

ب) $\sqrt{n-1} + \sqrt{y+1} = 3$

$$\sqrt{y+1} = 3 - \sqrt{n-1} \rightarrow y = (3 - \sqrt{n-1})^2 - 1$$

I) $n-1 \geq 0 \rightarrow n \geq 1$

II) $(3 - \sqrt{n-1})^2 \geq 0 \rightarrow \sqrt{n-1} \leq 3 \rightarrow n-1 \leq 9 \rightarrow n \leq 10$

$$D_f = I \cap II = [1, 10]$$

$f(n) = \frac{\log_e(n^2 - n - r)}{\sqrt{n^2 - 1} + 1}$

I) $n^2 - n - r > 0 \rightarrow (n-r)(n+1) > 0 \rightarrow \frac{-r}{1} < n < -1$

II) $n^2 - 1 > 0 \rightarrow n^2 > 1 \rightarrow n > 1 \text{ یا } n < -1$

III) $\sqrt{n^2 - 1} + 1 \neq 0 \rightarrow \sqrt{n^2 - 1} \neq -1$ همیشه برقرار است

$D_f = (-\infty, -1) \cup (r, +\infty)$

$I \cap II =$

۳

$f(n) = \sqrt{-n^2 + an + r} \rightarrow \text{دایره} \rightarrow \frac{-r}{-1} + \frac{b}{1} \rightarrow \left[\frac{-r}{1}\right] \text{ صدق می‌کند}$

$$0 = -1 - ra + r \rightarrow ra = r-1 \rightarrow a = \frac{r-1}{r}$$

$$\rightarrow \left[\frac{b}{1}\right] \rightarrow 0 = -b^2 - \frac{b}{r} + r \rightarrow b = \frac{\frac{1}{r} \pm \sqrt{\frac{1}{r^2} + 4r}}{-2} = -r \pm \sqrt{\frac{1}{r} + 4r^2}$$

$$a+b = \frac{-1}{r} + \frac{r}{r} = 1$$

۴

$g(n) = \sqrt{f(n) - n} \rightarrow f(n) \geq n$

I) $n-r \geq n \rightarrow -r \geq 0 \rightarrow n \geq 1$

II) $n+r \geq n \rightarrow r \geq 0 \rightarrow n \geq -r$

$$D_f = I \cap II = [1, +\infty)$$

۵

$$f(-r) = 3a - \varepsilon$$

$$f(a) = (a+1)(v) = 4a+v$$

$$2f(0) = f(-r) + a \rightarrow 12a + 1\varepsilon = 3a - \varepsilon + a \rightarrow 10a = -2\varepsilon \rightarrow a = -2/10$$

6

$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + r \left\{ \sqrt{\left(\frac{\sqrt{r-\sqrt{r}} + \sqrt{r+\sqrt{r}}}{2} \right)^2} = \sqrt{r-\sqrt{r}} + r + \sqrt{r} + r(\sqrt{\varepsilon-\sqrt{\varepsilon}}) = \sqrt{4} \right.$$

$$f(r-\sqrt{r}) + f\left(\frac{r+\sqrt{r}}{r-\sqrt{r}}\right) = \sqrt{r-\sqrt{r}} + \sqrt{r+\sqrt{r}} + \sqrt{r-\sqrt{r}} + \sqrt{\frac{r+\sqrt{r}}{r-\sqrt{r}}} + \varepsilon = 2(\sqrt{r-\sqrt{r}} + \sqrt{r+\sqrt{r}}) + \varepsilon$$

$$= 2\sqrt{4} + \varepsilon$$

7

$$\begin{aligned} n \rightarrow 2f(n) - 4f(-n) &= \varepsilon n^r + x \\ -n \rightarrow -3f(n) + 4f(-n) &= \varepsilon n^r + x \end{aligned} \rightarrow \begin{cases} 2f(n) - 4f(-n) = \varepsilon n^r + x \\ -3f(n) + 4f(-n) = \varepsilon n^r + x \end{cases}$$

$$\rightarrow -5f(n) = \varepsilon n^r + x \rightarrow f(n) = -\varepsilon n^r - \frac{x}{5}$$

8

$$x=0 \rightarrow 2f(0) - 0 = 3m-1 \xrightarrow{x^0}$$

$$n=-1 \rightarrow 0 + 4f(0) = 14 + 3m - 1 \xrightarrow{x^0} \left\{ \begin{aligned} 10f(0) &= 10m - 0 \\ 10f(0) &= 10m + \varepsilon 0 \\ 10f(0) &= \varepsilon 0 - 0f(0) = 0 \end{aligned} \right.$$

9

$$f(n) = an + b$$

$$\rightarrow an + b + \frac{a}{n} + b = \frac{3n^2 - 12n + 3}{n} \rightarrow an + \frac{a}{n} + 2b = 3n - 12 + \frac{3}{n}$$

$$\rightarrow a=3 \rightarrow f(n) = 3n - 9 \rightarrow f(-1) = -9 - 9 = -18$$

$$b = -9$$

10