نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \Rightarrow \frac{(x-1)^2-x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-2x+1}{x^2-x} \geq 0$

	$-\infty$		0		$\frac{1}{2}$		1		$+\infty$
$-2x+1$		$+$		$+$	0	$-$		$-$	
x^2-x		$+$	0	$-$		$-$	0	$+$	
$\frac{-2x+1}{x^2-x}$		$+$	‖	$-$	0	$+$	‖	$-$	

$D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\frac{1}{x+1}-\frac{2}{x(x+1)} \Rightarrow \frac{-x-1}{x^2+x}=\frac{(x^2+x-2)(-x-1)}{(2x+3)(x^2+x)} \Rightarrow (2x+3)(x^2+x)\neq 0 \Rightarrow x\neq -\frac{3}{2},-1,0$

$x^2+x-2\neq 0 \Rightarrow x\neq 1,-2$

$D_f=\mathbb{R}-\{-\frac{3}{2},-1,0,1\}$

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الف) $f(x)=\sqrt{\left[\left(\frac{1}{3}\right)^x-9\right](3^{2x}-4^x)} \Rightarrow \left[\left(\frac{1}{3}\right)^x-9\right](3^{2x}-4^x)\geq 0$

$\left(\frac{1}{3}\right)^x-9=0 \Rightarrow \left(\frac{1}{3}\right)^x=9 \Rightarrow x=-2$

$3^{2x}-4^x=0 \Rightarrow 3^{2x}=4^x \Rightarrow x=\frac{2x}{x} \Rightarrow x=2$

	$-\infty$		-2		2		$+\infty$
$\left(\frac{1}{3}\right)^x-9$		$+$	0	$-$		$-$	
$3^{2x}-4^x$		$-$		$+$	0	$-$	
$f(x)$		$+$	‖	$-$	0	$+$	

$D_f=(-\infty,-2]\cup[2,+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=2$ تابع ۲ $\Rightarrow \sqrt{y+1}=2-\sqrt[4]{x-1}$

$y=1-4\sqrt[4]{x-1}+\sqrt{x-1} \Rightarrow x-1\geq 0 \Rightarrow x\geq 1 \Rightarrow D_f=[1,+\infty)$

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$f(x)=\frac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1} \Rightarrow x^2-x-2>0 \Rightarrow (x-2)(x+1)>0 \Rightarrow (-\infty,-1)\cup(2,+\infty)$

$x^2-1+1>0 \Rightarrow x^2>0 \Rightarrow \mathbb{R}-\{0\}$

$\Rightarrow D_f=(-\infty,-1)\cup(2,+\infty)$

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$\sqrt{4+ax-x^2} \Rightarrow 4+ax-x^2\geq 0 \Rightarrow x^2-ax-4\leq 0 \quad \Delta=a^2-(4)(1)(-4)=a^2+16$

$x=\frac{a+\sqrt{a^2+16}}{2},\ \frac{a-\sqrt{a^2+16}}{2}$

$\Rightarrow [-2,b] \Rightarrow \frac{a-\sqrt{a^2+16}}{2}=-2 \Rightarrow a-\sqrt{a^2+16}=-4$

$\sqrt{a^2+16}=4+a$

$a^2+16=16+a^2+8a$

$8a+4=0$

$a=-\frac{1}{2}$

$a+b=-\frac{1}{2}-2=-\frac{5}{2}$

$-\frac{1}{2}+\frac{3}{2}=1$

$\Rightarrow a+b=1-\frac{3}{2}$

$\frac{-\frac{1}{2}+\sqrt{\frac{1}{4}+16}}{2}=b \Rightarrow 2b=-\frac{1}{2}+\sqrt{16+\frac{1}{4}}$

$2b+\frac{1}{2}=\sqrt{16+\frac{1}{4}} \Rightarrow 4b^2+\frac{1}{4}+2b=16+\frac{1}{4} \Rightarrow 4b^2+2b-16=0 \Rightarrow b=-2,\frac{3}{2}$

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$g(x)=\sqrt{f(x)-x} \Rightarrow f(x)-x\geq 0 \Rightarrow f(x)\geq x$

$f(x)=\begin{cases} 3x-1 & x\geq 1 \\ x+3 & x<1 \end{cases}$

$x\geq 1 \Rightarrow 3x-1\geq x \Rightarrow 2x-1\geq 0 \Rightarrow x\geq \frac{1}{2}$ ✓ $\mathbb{R}$

$x<1 \Rightarrow x+3\geq x \Rightarrow 3\geq 0$ ✓ $\mathbb{R}$

$\cap \Rightarrow D_f=[-3,+\infty)$

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$2f(\omega) - 2(a+1)(\omega) = 12a+12$ $\Rightarrow$ $12a+12 = 3a - 6 + a$

$f(-2) = 3a + 2(-2) = 3a - 4$ $\quad 10a = -18 \Rightarrow a = -1,8$

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$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$

$f(2-\sqrt{3}) + f(2+\sqrt{3})$

$\Rightarrow 4 + \left(\frac{1}{\sqrt{2+\sqrt{3}}} + \frac{1}{\sqrt{2-\sqrt{3}}}\right) + \left(\frac{\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}}{\sqrt{2}}\right) = 4 + \sqrt{2} + \sqrt{2} = 4 + 2\sqrt{2}$

$\left(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}\right) = A \rightarrow \left(\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}\right)^2 = A^2 \Rightarrow A^2 = \left(\sqrt{2+\sqrt{3}}\right)^2 + \left(\sqrt{2-\sqrt{3}}\right)^2 + 2\left(\sqrt{2+\sqrt{3}}\right)\left(\sqrt{2-\sqrt{3}}\right) =$

$2 + \sqrt{3} + 2 - \sqrt{3} + 2(1) = 4 = A^2 \Rightarrow A = \sqrt{4}$

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$2f(x) - 3f(-x) = 5x^2 - x$ $\quad f(x) = ax^2 + bx + c$

$f(-x) = ax^2 + bx + c$ $\Rightarrow$

$2(ax^2+bx+c) - 3(ax^2+bx+c) = 5x^2 - x$

$-ax^2 - bx - c = 5x^2 - x \Rightarrow \begin{cases} a = -5 \\ b = 1 \\ c = 0 \end{cases} \Rightarrow f(x) = -5x^2 + x$

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$(x+2)f(x) - 3xf(x+2) = 5x^2 - mx + 3m - 1$

$x = -2$, $f(-2)\times 0 + 2f(0) = 14 + 2m + 3m - 1$

$2f(0) = 13 + 5m$

$x = 0$, $2f(0) = 3m - 1$ $\Rightarrow$ $2f(0) = 3m - 1$

$2f(0) = 13 + 5m$

$8f(0) = 40$

$f(0) = \frac{40}{8}$

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$f(a) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x}$ $\Rightarrow$ $f(-1) + f(-1) = \frac{3+12+3}{-1} = -18$

$2f(-1) = -18$

$f(-1) = -9$

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$f(-1) = -a + b$ $\quad 2f(-1) = -2a + 2b$

$f\left(\frac{1}{-1}\right) = -a + b$

$-2a + 2b = \frac{3 - 12 + 3}{-1}$ $\quad 1,5$

$-2a + 2b = 6$

$-a + b = 3$

$f(-1) = -a + b$ $=$

$\Downarrow$

$f(-1) = 3$

$x = -x \rightarrow$ $2f(-x) - 2f(x) = 4x^2 + x$ (1

$2f(x) - 2f(-x) = 4x^2 - x$

$-3f(x) = 2x^2 + x \rightarrow f(x) = -\frac{4}{3}x^2 - \frac{1}{3}x$