

$$الف) f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \geq 0 \Rightarrow \frac{x-1}{x} - \frac{x^2}{x(x-1)} \geq 0 \Rightarrow \frac{-1}{x(x-1)} \geq 0 \Rightarrow \frac{1}{x-1} \geq 0 \Rightarrow x \geq 1$$

$x(x-1) = 0$ $\Rightarrow x=1, x=0$ نقطه بحرانی

$$ب) f(x) = \frac{1}{x+1} - \frac{1}{x(x+1)} - \frac{x-1}{x^2+x} = \frac{(x^2+x-1)(-x-1)}{(x+1)(x^2+x)} \geq 0 \Rightarrow \frac{(x^2+x-1)(-x-1)}{(x+1)(x^2+x)} \geq 0$$

$x^2+x-1=0 \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$ نقطه بحرانی

$$الف) f(x) = \sqrt{\left(\frac{1}{x}\right)^2 - 9} (3x - x^2) \geq 0 \Rightarrow \left[\left(\frac{1}{x}\right)^2 - 9\right] (3x - x^2) \geq 0$$

$\left(\frac{1}{x}\right)^2 - 9 = 0 \Rightarrow \frac{1}{x^2} = 9 \Rightarrow x = \pm \frac{1}{3}$ نقطه بحرانی

$$ب) \sqrt{x-1} + \sqrt{y+1} = 3 \Rightarrow y = 1 - 4\sqrt{x-1} + \sqrt{x-1} \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1 \Rightarrow D_f = [1, +\infty)$$

$$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow (-\infty, -1) \cup (2, +\infty)$$

$x^2 - 1 > 0 \Rightarrow x > 1 \Rightarrow \mathbb{R} - \{1\}$

$$\Rightarrow D_f = (-\infty, -1) \cup (2, +\infty)$$

$$\sqrt{3+ax-x^2} \Rightarrow 3+ax-x^2 \geq 0 \Rightarrow x^2-ax-3 \leq 0 \Rightarrow \Delta = a^2 - (4)(1)(-3) = a^2+12$$

$x = \frac{a \pm \sqrt{a^2+12}}{2}$ نقطه بحرانی

$$a+b = -\frac{1}{x} - y = -\frac{a}{x}$$

$-\frac{1}{x} + \frac{y}{x} = 1$

$$\Rightarrow a+b = \left[1, -\frac{a}{x}\right]$$

$$g(x) = \sqrt{f(x)-x} \Rightarrow f(x)-x \geq 0 \Rightarrow f(x) \geq x$$

$$f(x) = \begin{cases} 12x-4 & x \geq 1 \\ 12x+4 & x < 1 \end{cases}$$

$$x \geq 1 \Rightarrow 12x-4 \geq x = 11x-4 \geq 0 \Rightarrow x \geq \frac{4}{11} \Rightarrow D_f = \left[\frac{4}{11}, +\infty\right)$$

$$x < 1 \Rightarrow 12x+4 \geq x = 11x+4 \geq 0 \Rightarrow x \geq -\frac{4}{11}$$

$$f(a) - f(a+1)(v) = 15a + 15 \Rightarrow 15a + 15 = 15a - 15 + a$$

$$f(-1) = 15a + f(-2) = 15a - 15$$

$$1 \cdot a = -15 \Rightarrow a = -15$$

6

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + x$$

$$f(\sqrt{x}) + f(\sqrt{x}) = \left(\sqrt{\sqrt{x}} + \frac{1}{\sqrt{\sqrt{x}}} + \sqrt{x} \right) + \left(\sqrt{\sqrt{x}} + \frac{1}{\sqrt{\sqrt{x}}} + \sqrt{x} \right) = 2\sqrt{\sqrt{x}} + \frac{2}{\sqrt{\sqrt{x}}} + 2\sqrt{x}$$

7

$$(\sqrt{x+\sqrt{x}} + \sqrt{x-\sqrt{x}}) = A \Rightarrow (\sqrt{x+\sqrt{x}} + \sqrt{x-\sqrt{x}})^2 = A^2 \Rightarrow A^2 = (\sqrt{x+\sqrt{x}})^2 + (\sqrt{x-\sqrt{x}})^2 + 2(\sqrt{x+\sqrt{x}})(\sqrt{x-\sqrt{x}})$$

$$x + \sqrt{x} + x - \sqrt{x} + 2(\sqrt{x}) = y = A^2 \Rightarrow A = \sqrt{y}$$

$$f(x) - 3f(-x) = 15x - x \Rightarrow f(x) = ax^2 + bx + c$$

$$f(-x) = a(-x)^2 + b(-x) + c$$

8

$$f(ax^2 + bx + c) - 3f(-ax^2 - bx + c) = 15x^2 - x$$

$$-ax^2 - bx - c = 15x^2 - x \Rightarrow a = -15$$

$$b = 1$$

$$c = 0$$

$$\Rightarrow f(x) = -15x^2 + x$$

$$(x+y)f(x) - 4xf(x+y) = 15x^2 - 4mx + 4m - 1$$

$$f(-1)x + 4f(0) = 14 + 4m + 4m - 1$$

$$f(0) = 14 + 8m$$

$$f(0) = 14 + 8m$$

$$f(0) = 14 + 8m$$

$$f(0) = 14 + 8m$$

9

$$f(x) + f\left(\frac{1}{x}\right) = \frac{15x^2 - 15x + 1}{x} \Rightarrow f(-1) + f(-1) = \frac{15 - 15 + 1}{-1}$$

$$f(-1) = 1$$

$$f(-1) = 1$$

10

11

$$f(-1) = -a + b$$

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$$-a + b = \frac{15 - 15 + 1}{-1}$$

$$-a + b = 1$$

$$-a + b = 1$$

$$f(-1) = -a + b$$

12

$$f(-1) = 1$$