

20

مسئله 5: فرض کنید A مجموعه زیرمجموعه $\mathbb{R}$ باشد.

$$الف) f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \quad (1)$$

$$= \frac{1-2x}{x(x-1)} \rightarrow \frac{0 \quad 1/2 \quad 1}{+ \quad - \quad +} \quad D_f: (-\infty, 0) \cup [1/2, 1) \quad (5)$$

ب) $f(x) = \frac{1}{x+1} - \frac{x}{x}$ $\rightarrow$ $x \neq 0, -1, 1, -2$

$$\frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x(x+2) + (x-1)}{(x-1)(x+2)} = \frac{x^2 + 3x - 1}{(x-1)(x+2)} \rightarrow -0/2$$

$$D_f = \mathbb{R} - \{-2, -1, 0, 1, -0/2\}$$

ج) $f(x) = \sqrt{((1/x)^2 - 9)(x^2 - x^3)}$ $\rightarrow D_f: (-\infty, -2] \cup [2, 0, +\infty)$

$$x^2 - x^3 \geq 0 \rightarrow x^2(1-x) \geq 0 \rightarrow x \leq 1$$

$$(1/x)^2 - 9 \geq 0 \rightarrow \frac{1}{x^2} \geq 9 \rightarrow x^2 \leq 1/9 \rightarrow x \in [-1/3, 1/3]$$

$$D_f = (-\infty, -2] \cup [2, 0, +\infty)$$

د) $\sqrt{x-1} + \sqrt{y+1} = x \rightarrow \sqrt{y+1} = x - \sqrt{x-1}$ $\rightarrow x-1 \geq 0 \rightarrow x \geq 1$

فرض کنیم $\sqrt{y+1} = z$ $\rightarrow y+1 = z^2 \rightarrow y = z^2 - 1$

$x - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq x \rightarrow (x-1) \leq x^2 \rightarrow x^2 - x + 1 \geq 0$

$(1) \cap (2) \rightarrow [1, 1/2] = D_f$

$f(x) = \frac{\log x}{(x^2 - x - 2)}$ $\rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$

$$(-\infty, -1) \cup (2, +\infty) \quad (1)$$

$\sqrt{x^2 - 1} + 1 \rightarrow \sqrt{x^2 - 1} \neq -1 \checkmark$

$x^2 - 1 \geq 0 \rightarrow (-\infty, -1] \cup [1, +\infty) \quad (2)$

$(1) \cap (2) \rightarrow (-\infty, -1) \cup (2, +\infty)$

$$f(n) = \sqrt{r + an - n^2} \quad \begin{array}{l} n = r \text{ is a root} \\ \text{if } n = 0 \end{array} \rightarrow r - r a - \epsilon = 0$$

$$-ra = 1 \rightarrow a = -1/r$$

$$f(n) = \sqrt{-n^2 + n/r + r}$$

$$\omega_{\pm} = \frac{+1/r \pm \sqrt{1/r^2 - 4}}{-2} \rightarrow \begin{array}{l} +1/r \\ -r \end{array} \rightarrow \begin{array}{l} +r/r = b \\ -r \end{array} \rightarrow r/r - 1/r = 1 \quad \text{--- } a = 10$$

$$f(n) = \begin{cases} r_n - r; n \geq 1 \\ r_n + r; n \leq -1 \end{cases}$$

$$g(n) = \sqrt{f(n) - n} \rightarrow f(n) - n \geq 0 \rightarrow r_n - r - n \geq 0 \rightarrow$$

$$\begin{array}{l} n \leq -1 \rightarrow r_n + r - n \geq 0 \rightarrow n + r \geq 0 \rightarrow n \geq -r \\ n \geq 1 \rightarrow r_n - r - n \geq 0 \end{array}$$

$$\textcircled{1} \cap \textcircled{2} \rightarrow [-r, +\infty) = \text{IDP}$$

$$f(n) = \begin{cases} (a+1)(n+r); n \geq 1 \\ ra + rn; n \leq -1 \end{cases}$$

$$rf(a) = f(-r) + a$$

$$r(va, v) = ra - \epsilon + a \rightarrow 1\epsilon a, 1\epsilon \leq \epsilon a - \epsilon \rightarrow 1.a \leq -1\Lambda$$

$$\rightarrow a = -1/\Lambda$$

$$\gamma f(n) - \gamma f(-n) = \varepsilon n^r - n \xrightarrow{\gamma} \varepsilon f(n) - \gamma f(-n) = \lambda n^r - \gamma n$$

$$\gamma f(-n) - \gamma f(n) = \varepsilon n^r + n \xrightarrow{\gamma} \gamma f(-n) - \gamma f(n) = 1\gamma n^r + \gamma n$$

$$- \alpha f(n) = \gamma \cdot n^r + n$$

$$f(n) = -\varepsilon n - n/\alpha$$

$$(n+r)f(n) - \gamma f(n+r) = \varepsilon n^r - mn + \gamma m - 1$$

$$\underline{n=-r} \rightarrow \gamma f(\cdot) = 1\gamma + \gamma m + \gamma m - 1 \rightarrow \gamma f(\cdot) = \alpha m + 1\alpha = \alpha(m+r)$$

$$f(\cdot) = \alpha/\gamma (m+r)$$

$$\underline{n=0} \rightarrow \gamma f(\cdot) = \gamma m - 1 \Rightarrow \gamma m - 1/\gamma = \alpha/\gamma (m+r)$$

$$\rightarrow f(\cdot) = \frac{\gamma m - 1}{\gamma}$$

$$\gamma m - \gamma = \alpha m + 1\alpha$$

$$\varepsilon m = 1\lambda \rightarrow m = \frac{1}{\gamma}$$

$$\Rightarrow f(\cdot) = \frac{\gamma(\frac{1}{\gamma}) - 1}{\gamma} = \frac{\gamma \alpha/\varepsilon}{\gamma}$$

$$f(n) + f(\frac{1}{n}) = \frac{\gamma n^r - 1\gamma n + \gamma}{n} \xrightarrow{n=-1} f(-1) + f(-1) = \gamma + \gamma - 1$$

$$\gamma f(-1) = 1\lambda \rightarrow f(-1) = \frac{1}{\gamma}$$

$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + \gamma$$

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$$f(\sqrt{r}-\sqrt{r}) + f(\sqrt{r}+\sqrt{r}) = \sqrt{r}-\sqrt{r} + \frac{1}{\sqrt{r}-\sqrt{r}} + \gamma + \sqrt{r}+\sqrt{r} + \frac{1}{\sqrt{r}+\sqrt{r}} + \gamma$$

$$\downarrow$$

$$(\sqrt{r}+\sqrt{r})(\sqrt{r}-\sqrt{r}) = 1 \Rightarrow \sqrt{r}+\sqrt{r} = \frac{1}{\sqrt{r}-\sqrt{r}}$$

$$\rightarrow \sqrt{r}-\sqrt{r} + \sqrt{r}+\sqrt{r} + \gamma + \sqrt{r}-\sqrt{r} + \sqrt{r}-\sqrt{r} + \gamma = \underbrace{\gamma(\sqrt{r}-\sqrt{r} + \sqrt{r}+\sqrt{r}) + 2\gamma}_P$$

$$\textcircled{5} \quad (\sqrt{2-\sqrt{2}} + \sqrt{2+\sqrt{2}})^2 = 2 - \cancel{\sqrt{2}} + 2\sqrt{2} + 2\sqrt{(2-\cancel{\sqrt{2}})(2+\sqrt{2})} = 4$$

$$\Rightarrow P = 2(\sqrt{4} + 2) = \boxed{2\sqrt{4} + 4}$$