حسابان یازدهم دفتر A ، الف شماره ۴، ۵

1) الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \to \frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{(x-1)^2-x^2}{x(x-1)}$

$=\frac{1-2x}{x(x-1)} \to$ ریشه‌ها: $x=\frac{1}{2}$ ، $x=0$ ، $x=1$

	0		$\frac{1}{2}$		1	
$+$		$-$		$+$		$-$

$D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \to$ مخرج ها $\neq 0 \to x\neq 0,-1,1,-2$

$\frac{3}{x-1}+\frac{1}{x+2}\neq 0 \to \frac{3x+6+x-1}{(x+1)(x+2)}=\frac{4x+5}{(x+1)(x+2)} \to -\frac{5}{4}$

$D_f=\mathbb{R}-\{-2,-1,0,1,-\frac{5}{4}\}$

2) الف) $f(x)=\sqrt{\left((\frac{1}{3})^x-9\right)\left(32-4^x\right)} \to P\geq 0$

	-2		2.5	
$+$		$-$		$+$

$D_f=(-\infty,-2]\cup[2.5,+\infty)$

$3^{-x}=3^2 \to x=-2 \qquad 2^5=2^{2x} \to x=2.5 \to (-\infty,-2]\cup[2.5,+\infty)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \to \sqrt{y+1}=3-\sqrt[4]{x-1} \to x-1\geq 0 \to x\geq 1$ (1)

خاطر $\sqrt{y+1}$ همیشه بزرگتر مساوی صفره

$3-\sqrt[4]{x-1}\geq 0 \to \sqrt[4]{x-1}\leq 3 \xrightarrow{(\;)^4} x-1\leq 81 \to x\leq 82$ (2)

$(1)\cap(2) \to [1,82]=D_f$

3- $f(x)=\frac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1} \to x^2-x-2>0 = (x-2)(x+1)$

	-1		2	
$+$		$-$		$+$

$(-\infty,-1)\cup(2,+\infty)$ (1)

$\sqrt{x^2-1}+1 \to \sqrt{x^2-1}\neq -1$ ✓ همیشه صحیح

$x^2-1\geq 0 \to (-\infty,-1]\cup[1,+\infty)$ (2)

$(1)\cap(2) \to (-\infty,-1)\cup(2,+\infty)$

DAT

۴

$$f(x)=\sqrt{3+ax-x^2}\ \xrightarrow[\text{جواب معادله}]{x=2\ \text{صفر تابع}}\ 3-2a-4=0$$

$$\boxed{-2a=1 \to a=-1/2}$$

$$f(x)=\sqrt{-x^2+x/2+3}$$

ریشه‌ها :
$$\frac{+1/2 \pm \sqrt{\cancel{49/4}}^{\,7/2}}{-2} \begin{cases} \to +3/2 = b \\ \to -2 \end{cases} \longrightarrow 3/2-1/2 = 1 = a+b$$

۵_

$$f(x)=\begin{cases} 3x-2 \ ; \ x\geq 1 \\ 2x+3 \ ; \ x<1 \end{cases}$$

$$g(x)=\sqrt{f(x)-x} \to f(x)-x\geq 0 \xrightarrow{x\geq 1} 3x-2-x\geq 0 \to 2x-2\geq 0 \quad x\geq 1 \ \ (1)$$

$$\xrightarrow{x<1} 2x+3-x\geq 0 \to x+3\geq 0 \to x\geq -3 \ \ (2)$$

$$(1)\cap(2) \to [-3,+\infty)=D_f$$

۶_

$$f(x)=\begin{cases} (a+1)(x+2) \ ; \ x>1 \\ 3a+2x \ ; \ x\leq 1 \end{cases}$$

$$2f(5)=f(-2)+a$$

$$2(7a+7)=3a-4+a \to 14a+14=4a-4 \to 10a=-18$$

$$\to \boxed{a=-1/8}$$

۸-

$$2f(x) - 3f(-x) = 4x^2 - x \xrightarrow{\times 2} 4f(x) - 6f(-x) = 8x^2 - 2x$$

$$2f(-x) - 3f(x) = 4x^2 + x \xrightarrow{\times 3} 6f(-x) - 9f(x) = 12x^2 + 3x$$

$$-5f(x) = 20x^2 + x$$

$$\boxed{f(x) = -4x - x/5}$$

۹-

$$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$$

$$\xrightarrow{x=-2} 6\,f(0) = 16 + 2m + 3m - 1 \to 6f(0) = 5m + 15 = 5(m+3)$$

$$f(0) = 5/6\,(m+3)$$

$$\xrightarrow{x=0} 2f(2) = 3m - 1 \Rightarrow 3m - 1/2 = 5/6\,(m+3)$$

$$\to f(2) = \frac{3m-1}{2} \qquad 9m - 3 = 5m + 15$$

$$4m = 18 \to \boxed{m = 9/2}$$

$$\Rightarrow f(2) = \frac{3(9/2) - 1}{2} = \boxed{25/4}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \xrightarrow{x=-1} f(-1) + f(-1) = 3 + \cancel{12} + 3 - 1.$$

$$2f(-1) = 18 \to \boxed{f(-1) = 9}$$

۷-

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad , \qquad \sqrt{2+\sqrt{3}} \qquad \sqrt{2-\sqrt{3}}$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{\cancel{1}}{\cancel{\sqrt{2-\sqrt{3}}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{\cancel{1}}{\cancel{\sqrt{2+\sqrt{3}}}} + 2$$

$$(2+\sqrt{3})(2-\sqrt{3}) = 1 \Rightarrow 2+\sqrt{3} = \frac{1}{2-\sqrt{3}}$$

$$\to \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2 + \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 2 = 2\underbrace{\left(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + 2\right)}_{P}$$

$$\left(\sqrt{٢-\sqrt{٣}}+\sqrt{٢+\sqrt{٣}}\right)^{٢} = ٢-\cancel{\sqrt{٣}}+٢+\cancel{\sqrt{٣}}+٢\sqrt{\cancel{(٢-\sqrt{٣})(٢+\sqrt{٣})}}_{١} = ٦$$

$$\Rightarrow ٦P = ٢(\sqrt{٦}+٢) = \boxed{٢\sqrt{٦}+٤}$$