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الف) $x \neq 0$
 $\frac{x-1}{x} - \frac{x}{x-1} \geq \frac{(x-1)^2 - x^2}{x(x-1)} = \frac{x^2 - 2x + 1 - x^2}{x(x-1)} = \frac{-2x+1}{x(x-1)} \geq \sqrt{\frac{-2x+1}{x(x-1)}}$
 $\frac{-2x+1}{x(x-1)} \geq 0 \Rightarrow \frac{1}{x+1} \rightarrow x \neq -1$
 $\frac{x}{x-1} \rightarrow x \neq 1$
 $R = \{-1, -1, 0, 1\}$
 $\frac{1}{x-1} + \frac{1}{x+1} \neq 0 \Rightarrow x \neq -\frac{1}{2}$

الف) $(-\frac{1}{x})^x (x-2)^x \geq 0 \Rightarrow x-2 \leq 0 \Rightarrow x \leq 2$
 $\log_{\frac{1}{x}} x \leq x$
 $x \geq 2 \Rightarrow \frac{1}{x} \leq \frac{1}{2} \Rightarrow \log_{\frac{1}{x}} x \geq \log_{\frac{1}{2}} x$
 $x \leq 2 \Rightarrow \frac{1}{x} \geq \frac{1}{2} \Rightarrow \log_{\frac{1}{x}} x \leq \log_{\frac{1}{2}} x$
 $x = 2, a \rightarrow 2^2 = 4 \rightarrow \sqrt{4} = 2 \rightarrow x \geq \frac{a}{2} \Rightarrow x \geq 2$
 ب) $x-1 \geq 0 \Rightarrow x \geq 1$
 $y+1 \geq 0 \Rightarrow y \geq -1$
 $a = \sqrt{x-1} \Rightarrow x = a^2 + 1$
 $b = \sqrt{y+1} \Rightarrow y = b^2 - 1$
 $a+b=3 \Rightarrow b=3-a$
 $y = (3-a)^2 - 1$
 $x = a^2 + 1$

$\log_{\frac{1}{x}} (x^2 - x - 2) \geq 0 \Rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0$
 $\frac{-1}{x-2} + \frac{1}{x+1} \geq 0 \Rightarrow \frac{-x+1+x+1}{(x-2)(x+1)} \geq 0 \Rightarrow \frac{2}{(x-2)(x+1)} \geq 0$
 $\sqrt{x^2-1} + 1 \neq 0 \Rightarrow \sqrt{x^2-1} \neq -1$
 $\sqrt{x^2-1} \geq 0 \Rightarrow \sqrt{x^2-1} \geq -1$
 $(-\infty, -1] \cup [2, +\infty)$

$3-2a-x \geq 0 \Rightarrow -(2a+x) \geq -3 \Rightarrow 2a+x \leq 3$
 $x+ba-b^2 \geq 0 \Rightarrow -(b^2-ba) \geq -3 \Rightarrow b^2-ba \leq 3$
 $-ba+b^2 = 3 \Rightarrow \frac{1}{2}b + \frac{1}{2} = 3 \Rightarrow \frac{1}{2}b = \frac{5}{2} \Rightarrow b = 5$
 $a+b = \frac{1}{2} = 2 \Rightarrow a = -\frac{3}{2}$
 $b = \frac{1}{2} \Rightarrow a+b=1$

$f(1) = 3-2=1 \Rightarrow g(x) = 0 \checkmark$
 $f(0) = 3 \Rightarrow g(x) = \sqrt{3} \checkmark$
 $f(-1) = 1 \Rightarrow g(x) = \sqrt{1} \checkmark$
 $f(-3) = 0 \Rightarrow g(x) = \sqrt{0} \checkmark$
 $f(-5) = -2 \Rightarrow g(x) = \sqrt{-2} \checkmark$
 $D_g = [-3, +\infty)$

$$f(x) = (a+1)(x+1) \rightarrow \forall a+1 \quad f(x) = (a+1)x + (a+1)$$

$$f(-1) = a-2 \quad f(x) = f(-1) + a \rightarrow (a+1)x = a-2 + a$$

$$a = -1 \quad a = -\frac{1}{1} \quad a = -1$$

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$$f(x-\sqrt{x}) = \sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}} + x \quad f(x+\sqrt{x}) = \sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}} + x$$

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$$\left(\sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}}\right) + \left(\sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}}\right) + x = f(x-\sqrt{x}) + f(x+\sqrt{x})$$

$$\frac{(x-\sqrt{x})(x+\sqrt{x})}{(x-\sqrt{x})(x+\sqrt{x})} = 1$$

$$x(\sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}}) + x \rightarrow x\sqrt{4} + x$$

-x

$$f(x) - f(-x) = 5x^2 - x \quad f(x) + f(-x) = -1x^2 \Rightarrow f(x) - f(-x) = -1x^2 - x$$

$$f(x) + f(-x) = -1x^2 \quad f(-x) = -1x^2 - f(x) \Rightarrow f(x) - (-1x^2 - f(x)) = -1x^2 - x$$

$$2f(x) = -2x^2 - x \rightarrow f(x) = -x^2 - \frac{x}{2}$$

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$$\Rightarrow \Rightarrow (1+x)f(0) - 3x \times f(1) = 5x^2 - m \times 0 + m+1 \quad f(0) = m+1 \quad f(1) = \frac{m+1}{1}$$

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$$x = -1 \rightarrow f(0) = \frac{10 + 3m}{4}$$

$$\frac{10 + 3m}{4} = \frac{m+1}{1}$$

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$$\rightarrow m = \frac{4}{7} \quad f(0) = \frac{10}{7}$$

$$f(x) = ax + b \rightarrow f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$\frac{a}{x} + b + ax + b = \frac{a + ax^2 + bx}{x}$$

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$$\Rightarrow \frac{ax^2 + bx + a}{x}$$

$$b = -1 \quad b = -4 \quad a = 1$$

$$f(-1) = -a + b \rightarrow -a + b = -1 - 4 = -5$$

$$x-1 \rightarrow x \geq 1$$

$$x = \sqrt{x-1} + \sqrt{y+1} \rightarrow x \leq 1 \quad D_f = [1, \infty)$$

(-1)