

$$f(x) = (a+1)(x+1) \rightarrow \forall a \in \mathbb{R} \quad f(x) = (a+1)x + (a+1)$$

$$f(-1) = a - 1 \quad f(x) = f(-1) + a \rightarrow (a+1)x + (a+1) = a - 1 + a$$

$$a = -1 \quad a = -\frac{1}{1} \quad a = -1 \quad \left. \vphantom{\begin{matrix} a = -1 \\ a = -\frac{1}{1} \\ a = -1 \end{matrix}} \right\}$$

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$$f(x-\sqrt{x}) = \sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}} + x \quad f(x+\sqrt{x}) = \sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}} + x$$

$$\left(\sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}} \right) + \left(\sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}} \right) + x = f(x-\sqrt{x}) + f(x+\sqrt{x})$$

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$$\begin{aligned} f(x) - f(-x) &= 5x^2 - x & + & \quad x^2 \\ \Rightarrow f(x) + f(-x) &= -11x^2 & \Rightarrow & \quad f(x) - f(-x) = -11x^2 - x \\ f(-x) - f(x) &= 5x^2 + x & f(x) - f(-x) &= 2x^2 - x \end{aligned}$$

$$f(x) + f(-x) = -11x^2$$

$$f(x) - f(-x) = -11x^2 - x \Rightarrow f(-x) = -11x^2 - f(x) \Rightarrow f(x) - (-11x^2 - f(x)) = -11x^2 - x$$

$$\Rightarrow f(x) = -5.5x^2 - x \rightarrow f(x) = -5.5x^2 - \frac{x}{1}$$

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$$\Rightarrow \Rightarrow (1+x)f(0) - 3x \cdot f(1) = 5x^0 - m x^0 + m+1 \quad f(0) = m+1 \quad f(1) = \frac{m+1}{1}$$

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$$f(x) = ax + b \rightarrow f\left(\frac{1}{x}\right) = \frac{a}{x} + b$$

$$\frac{a}{x} + b + ax + b = \frac{a + ax^2 + bx}{x}$$

$$\Rightarrow \frac{ax^2 + bx + a}{x}$$

$$\begin{aligned} b &= -1 \\ b &= -4 \\ a &= 3 \end{aligned}$$

$$f(-1) = -a + b \rightarrow -a + b = -3 - 4 = -7$$

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