$$f(x)=\begin{cases}(a+1)(x+2) & x>1\\ 3a+2x & x\le 1\end{cases}$$

$$2f(5)=f(-2)+a$$

۶ جواب

$$f(5)=(a+1)(5+2)=7a+7$$
$$f(-2)=3a-4$$
$$\rightarrow 2(7a+7)=3a-4+a \rightarrow 14a+14=3a-4+a$$
$$10a=-18 \rightarrow \boxed{a=-\frac{18}{10}}$$

۷

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2 \rightarrow \sqrt{2+\sqrt3}+\frac{1}{\sqrt{2+\sqrt3}}+2+\sqrt{2-\sqrt3}+\frac{1}{\sqrt{2-\sqrt3}}+2=2\left(\sqrt{2+\sqrt3}+\sqrt{2-\sqrt3}\right)+4$$

$$\boxed{2\sqrt6+4}$$ جواب

$$f(2-\sqrt3)+f(2+\sqrt3)$$

$$\frac{1}{\sqrt{2+\sqrt3}\times(\sqrt{2-\sqrt3})}=\frac{\sqrt{2-\sqrt3}}{\sqrt{4-3}},\quad \frac{1}{\sqrt{2-\sqrt3}}\Rightarrow\sqrt{2+\sqrt3}$$

$$\left(\sqrt{2+\sqrt3}+\sqrt{2-\sqrt3}\right)=\sqrt6$$
$$\left(\sqrt{2+\sqrt3}+\sqrt{2-\sqrt3}\right)^2=2+\cancel{\sqrt3}+2-\cancel{\sqrt3}+2\sqrt{4-3}=6$$

۸

$$2f(x)-3f(-x)=4x^2-x$$

$$\boxed{-4x^2+\frac{1}{5}x}$$ جواب

$$x\rightarrow 2f(x)-3f(-x)=4x^2-x \xrightarrow{\times 2} 4f(x)-6f(-x)=8x^2-2x$$
$$-x\rightarrow 2f(-x)-3f(x)=4x^2+x \xrightarrow{\times 3} 6f(-x)-9f(x)=12x^2+3x$$
$$-5f(x)=20x^2+x \rightarrow f(x)=-\frac{20x^2+x}{5}$$

۹

$$(x+2)f(x)-3xf(x+2)=4x^2-mx+3m-1$$

$$f(0)=\frac{25}{4}$$ جواب

$$x=0\rightarrow (2)f(0)=3m-1 \rightarrow 2f(0)=3m-1 \xrightarrow{\times 3} 6f(0)=9m-3$$
$$6f(0)=15+5m$$
$$x=-2\rightarrow +6f(0)=16+2m+3m-1 \rightarrow 6f(0)=16+5m-1$$
$$4m+18=0 \rightarrow 4m=18 \rightarrow m=\frac{18}{4}=\boxed{\frac{9}{2}}$$

m

$$2f(0)=3\times\frac{9}{2}-1 \Rightarrow f(0)=\frac{\frac{25}{2}}{2}=\frac{25}{4}$$

۱۰

$$f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x}$$

$$ax+b+a\left(\frac{1}{x}\right)+b=\frac{3x^2-12x+3}{x}$$
$$ax+b+\frac{a}{x}+b=\frac{3x^2-12x+3}{x}$$
$$ax+2b+\frac{a}{x}=\frac{3x^2-12x+3}{x}$$
$$ax+2b+\frac{a}{x}=3x-12+\frac{3}{x}$$
$$a=3,\ b=-6$$
$$f(x)=ax+b \rightarrow f(x)=3x-6 \rightarrow f(-1)=\boxed{-3-6=-9}$$

راه دوم

$$f(-1)$$
$$x=-1 \rightarrow f(-1)+f(-1)=\frac{3+12+3}{-1}$$
$$\rightarrow 2f(-1)=-18 \rightarrow \boxed{f(-1)=-9}$$

نام و نام خانوادگی: بهار شهیدان — تکلیف ۴ — گروه یازدهم دختر A

۱

الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$\frac{x^2+1-2x-x^2}{x^2-x}=\frac{1-2x}{x^2-x}\geq 0$

$\frac{1}{2}$ ، 0 ، 1

$+\ \ 0\ \ -\ \ \frac{1}{2}\ \ +\ \ 1\ \ -$

$\boxed{D=(-\infty,0)\cup[\frac{1}{2},1)}$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$

$x+1\neq 0\to x\neq -1$

$x\neq 0$

$x-1\neq 0\to x\neq 1$

$x+2\neq 0\to x\neq -2$

$\frac{3x+6+x-1}{x^2+x-2}\neq 0\to 4x-5\neq 0\to x\neq -\frac{5}{4}$

$\boxed{D=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}}$

۲

$f(x)=\sqrt{\left(\left(\frac{1}{3}\right)^x-9\right)\left(32-4^x\right)}$

$\left(\left(\frac{1}{3}\right)^x-9\right)\left(32-4^x\right)\geq 0$

$4^x=32\to 2^{2x}=\cancel{32}\ 2^5\to 2x=5\to x=\frac{5}{2}$

$\left(\frac{1}{3}\right)^x=9$

$\left((3)^{-1}\right)^x=3^2\to 3^{-x}=3^2\to -x=2\to x=-2$

-2 ، $\frac{5}{2}$

$+\ \ -2\ \ -\ \ \frac{5}{2}\ \ +$

$\boxed{D=(-\infty,-2]\cup[\frac{5}{2},\infty)}$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt{y+1}=-\sqrt[4]{x-1}+3$

$3-\sqrt[4]{x-1}\geq 0\to 3\geq\sqrt[4]{x-1}$

توان ۴: $81\geq x-1\to x\leq 82$ ①

$\sqrt[4]{x-1}\to x-1\geq 0\to x\geq 1$ ②

$\boxed{D_1\cap D_2=[1,82]}$

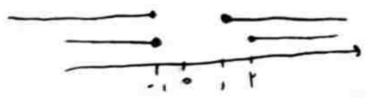
۳

$f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x^2-x-2>0\to(x-2)(x+1)>0$ $\quad +\ \ -1\ \ -\ \ 2\ \ +$

$\sqrt{x^2-1}+1\neq 0\to\sqrt{x^2-1}\neq -1$ ✓ ← هیچ وقت صفر نخواهد شد

$x^2-1\geq 0\to x^2\geq 1\to x\geq 1\quad x\leq -1$



$D=(-\infty,1]\cup[2,\infty)$

۴

$\sqrt{3+ax-x^2}$

① $x_1=-2, x_2=b$ ریشه ها $\leftarrow D=[-2,b]$

$-x^2+ax+3\geq 0$

$a+b=$

① $\begin{cases}S=x_1+x_2=-\frac{b}{a}\to b-2=-\frac{a}{-1}\to b-2=a\\ P=x_1\times x_2=\frac{c}{a}\to -2\times b=\frac{3}{-1}\to 2b=3\to b=\frac{3}{2}\end{cases}\to a=\frac{3}{2}-2=-\frac{1}{2}\to a+b=-\frac{1}{2}+\frac{3}{2}=1$

۵

$f(x)=\begin{cases}3x-2 & x\geq 1\\ 2x+3 & x<1\end{cases}$

① $x\geq 1\to 3x-2\geq x\to 2x\geq 2\to x\geq 1$

② $x<1\to 2x+3\geq x\to x\geq -3\Rightarrow -3\leq x<1$

$D_g\to(x\geq 1)\cup(-3\leq x<1)\to x\geq -3$

$g(x)=\sqrt{f(x)-x}\to f(x)\geq x$

$D_g=[-3,\infty)$