الف) $f(x) = \sqrt{\dfrac{x-1}{x} - \dfrac{x}{x-1}} \rightarrow$ ①

$$\frac{x-1}{x} - \frac{x}{x-1} \rightsquigarrow$$

$$\frac{(x-1)^2 - x^{\rightsquigarrow}}{x(x-1)} = \frac{1-2x}{x(x-1)}$$

	0		$1/2$		1	
$+$		$-$		$+$		$-$

$D = (-\infty, 0) \cup [1/2, 1)$

ب) $$\frac{\dfrac{1}{x+1} - \dfrac{2}{x}}{\dfrac{3}{x-1} + \dfrac{1}{x+2}} \rightsquigarrow \frac{3(x+2)}{(x+2)(x-1)} + \frac{1(x-1)}{(x+2)(x-1)}$$

$$\frac{3x+6+x-1}{(x+2)(x-1)} \Rightarrow 4x+5=0 \quad x \neq -\frac{5}{4}$$

$$\boxed{\mathbb{R} - \{0, -1, 1, -2, -5/4\}}$$

باران حاتمی

date: .... Sub: ....

$a+b = \frac{3}{2} - \frac{1}{2} = \frac{2}{2} = 1$

۵

$$f(x) \begin{cases} 3x-2 \; ; & x \geq 1 \\ 2x+3 \; ; & x < 1 \end{cases} \qquad g(x) = \sqrt{f(x)-x}$$

$f(x) - x \geq 0 \rightarrow f(x) \geq x$

$3x-2-x = 2x-2 \geq 0 \rightarrow 2x \geq 2 \rightarrow x \geq 1$

$2x+3-x = x+3 \geq 0 \rightarrow x \geq -3$

$\xrightarrow{\cap} D = [-3, +\infty)$

۶

$$f(x) \begin{cases} (a+1)(x+2) \; ; & x \geq 1 \\ 3a+2x \; ; & x < 1 \end{cases}$$

$2f(5) = f(-2)+a$ با شرط مقدار a را بدست آورید

$f(5) = (a+1)(5+2) = 7(a+1) = 7a+7$

$f(-2) = 3a+2(-2) = 3a-4$

$2(7a+7) = 3a-4+a \rightarrow 14a+14 = 3a-4+a$

$10a = -18 \rightarrow a = \frac{-18}{10}$

$a = -\frac{9}{5}$

باران صائم

date: .... Sub: ....

۳) دامنه‌ی تابع با ضابطه‌ی $f(x) = \dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$ را بدست آورید

$\sqrt{x^2-1} \geq 0$ ؛ $\sqrt{x^2-1} \geq 1$ ؛ $x^2 - \begin{cases} x \geq 1 \\ \text{یا} \\ x \leq -1 \end{cases}$

$\sqrt{x^2-1}+1 \neq 0$

$\sqrt{x^2-1} \neq -1$

$x^2-x-2 > 0$

$(x-2)(x+1) > 0$

$x-2>0$ یا هر دو + $\rightarrow x>2$

$x+1<0$ یا هر دو منفی $\rightarrow x<-1$

$x^2-1 \geq 0$

$x^2 \geq 1$

$x \geq 1$ یا $x \leq -1$

مخرج: $x \geq 1$ یا $x \leq -1$

لگاریتم: $x > 2$ یا $x < -1$

$D_f = (-\infty, -1) \cup (2, +\infty)$

---

۴) $\sqrt{3+ax-x^2}$ $\quad [-2, b] \quad a+b=?$

$x = \dfrac{a \pm \sqrt{a^2+12}}{2} \rightarrow x_{min} = \dfrac{a-\sqrt{a^2+12}}{2} = -2$

$a - \sqrt{a^2+12} = -4 \rightarrow \sqrt{a^2+12} = a+4$

$a^2+12 = (a+4)^2 \rightarrow \cancel{a^2}+12 = \cancel{a^2}+8a+16$

$\boxed{8a+4 \Rightarrow 8a=-4 \rightarrow a=-\tfrac{1}{2}}$

$\dfrac{a+\sqrt{a^2+12}}{2} = \dfrac{-\frac{1}{2}+\sqrt{(-\frac{1}{2})^2+12}}{2}$

$= -0/5 + 3/5$ $\quad \frac{3}{2} = 1/5 = b$

پیکو

۴

$$f(x) = \sqrt{\left[\left(\frac{1}{3}\right)^x - 9\right](32 - 4^x)}$$

$$(3^{-x} - 9)(32 - 4^x) \geqslant 0$$

$$3^{-x} - 9 = 0 \longrightarrow 3^{-x} = 3^2 \longrightarrow -x = 2 \longrightarrow \underline{x = -2}$$

$$32 - 4^x = 0 \longrightarrow 4^x = 32 \longrightarrow 2^{2x} = 2^5$$

$$2x = 5 \longrightarrow x = 5/2$$

$$D = (-\infty, -2] \cup [5/2, +\infty)$$

$$\sqrt[4]{x-1} + \sqrt{y+1} = 3 \qquad x - 1 \geqslant 0$$

$$x \geqslant 1$$

$$y + 1 \geqslant 0 \longrightarrow y \geqslant -1$$

$$\sqrt{y+1} = 3 \longrightarrow y = 8$$

$$\sqrt[4]{x-1} = 3 \longrightarrow x = 82$$

$$0 \leqslant \sqrt[4]{x-1} \leqslant 3 \longrightarrow 1 \leqslant x \leqslant 82$$

$$D = [1, 82]$$

$s=\sqrt{a}$, $t=\sqrt{b}$

date: $t=\frac{1}{s}$ $\leftarrow f(\underbrace{۲-\sqrt{۳}}_{a}) + f(\underbrace{۲+\sqrt{۳}}_{b}) = ?$ حاصل را بیابید

$s+t = s+\frac{1}{s}$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + ۲ \qquad a^۲+b^۲=۴$$ ۷

$$ab = (۲-\sqrt{۳})(۲+\sqrt{۳}) = ۴-۳ = ۱$$

$$f(a)+f(b) = (s+\frac{1}{s}+۲)+(t+\frac{1}{t}+۲) =$$

$$(s+t)+(\frac{1}{s}+\frac{1}{t})+۴$$

$$f(a)+f(b) = ۲(s+\frac{1}{s})+۴ \qquad s^۲=a \;, t^۲=b$$

$$(s+t)^۲ = s^۲+۲st+t^۲ = a+۲+b$$

$$(۲-\sqrt{۳})+۲+(۲+\sqrt{۳}) = ۶$$

$$s+\frac{1}{s} = \sqrt{۶} \qquad f(۲-\sqrt{۳})+f(۲+\sqrt{۳}) =$$

$$\boxed{۴+۲\sqrt{۶}}$$

۸

$$۲f(x) - ۳f(-x) = ۴x^۲ - x$$

$$\begin{cases} ۲f(-x) - ۳f(x) = ۴x^۲+x \\ ۲f(x) - ۳f(-x) = ۴x^۲-x \end{cases}$$

$$f(x) = -۴x^۲ - \frac{x}{۵}$$

$$f(-x) = -۴x^۲ + \frac{x}{۵}$$

باران صالحی

date: $f(0)$?

Sub:

9

$$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$$

$$(0+2)f(0) - 3(0)\cancel{f(0+2)} = \cancel{4(0)^2} - x(0) + 3m - 1$$

$$2f(0) - 0 = 3m - 1 \longrightarrow \boxed{f(0) = \frac{3m-1}{2}}$$

10

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \qquad f(-1) = ?$$

$$(ax+b) + \left(a\frac{1}{x} + b\right) = ax + \frac{a}{x} + 2b$$

$$f(x) + f\left(\frac{1}{x}\right) = 3x - 12 + \frac{3}{x} \longrightarrow$$

$$a = 3 \quad 2b = -12 \longrightarrow b = -6$$

پس $\longrightarrow$ $f(x) = 3x - 6 \longrightarrow f(-1) = 3(-1) - 6 =$

$$-3 - 6 = \boxed{-9}$$