14,75

نگار صمدانی

الف) $f(x) = \sqrt{\dfrac{x-1}{x} - \dfrac{x}{x-1}}$

$x \neq 0 \quad x \neq 1 \quad , \quad \dfrac{x-1}{x} - \dfrac{x}{x-1} \geq 0$

$\dfrac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \qquad \dfrac{-2x+1}{x(x-1)} \geq 0$

| $x$ | | $0$ | | $\frac{1}{2}$ | | $1$ | |
|---|---|---|---|---|---|---|---|
| $-2x+1$ | $+$ | | $+$ | $\cdot$ | $-$ | | $-$ |
| $x(x-1)$ | $-$ | | $+$ | | $+$ | $\cdot$ | $-$ |
| | $-$ | | $+$ | | $-$ | | $+$ |
| | $+$ | | $-$ | | $+$ | | |

$D_f \ (0, \tfrac{1}{2}] \cup (1, +\infty)$

1,5

ب) $f(x) = \dfrac{\dfrac{1}{x+1} - \dfrac{2}{x}}{\dfrac{2}{x-1} + \dfrac{1}{x+2}}$

$x \neq 0 \quad x = -1 \quad , x \neq 1 \quad x \neq -2 \qquad x = -\dfrac{5}{4}$

$f(x) = \dfrac{\dfrac{-x+2}{(x+1)x}}{\dfrac{4x+5}{(x-1)(x+2)}} = \dfrac{(x-1)(x+2)(2-x)}{(4x+5)(x+1)x}$

$R - [-2, -\tfrac{5}{4}, -1, 0, 1]$

(2)

الف) $f(x) = \sqrt{\left(\left(\tfrac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right)}$

1,25

$(3^{-x} - 9)(2^5 - 2^{2x}) \geq 0$

| | | $-2$ | | $2,5$ | |
|---|---|---|---|---|---|
| $(3^{-x}-9)$ | $+$ | $\cdot$ | $-$ | | $-$ |
| $2^5 - 2^{2x}$ | $+$ | | $+$ | $\cdot$ | $-$ |
| | $+$ | $\cdot$ | $-$ | $\cdot$ | $+$ |

$D_f \ (+\infty \ -2] \cup [2,5, +\infty)$

۲ ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt[4]{(x-1)}-3=\sqrt{y+1} \longrightarrow y=\left[\sqrt[4]{(x-1)}-3\right]^2-1$

$D_f\ [1,+\infty)$

$D_f=(-\infty,-1)\cup(2,+\infty)$

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۳)

$f(x)=\dfrac{\log_{4}(x^2-x-2)}{\sqrt{x^2-1}+1}$

$x\geq 1,\ x\leq -1$

① $\sqrt{x^2-1} \longrightarrow x^2-1\geq 0 \longrightarrow -1\leq \cancel{x} \leq 1$

② $x^2-x-2>0 \longrightarrow$

$(x+1)(x-2)$

| | $-1$ | | $2$ | |
|---|---|---|---|---|
| $+$ | $0$ | $-$ | $0$ | $+$ |

۴)

$f(x)=\sqrt{3+ax-x^2} \qquad D_f[-2,b]$

$-2, b$ ریشه‌ها $\longrightarrow \underset{x_1x_2}{-2b=3} \longrightarrow b=+1{,}5$

۲

$x_2=-2 \longrightarrow 3-2a-4=0 \longrightarrow a=-\frac{1}{2}$

$a+b=-\frac{1}{2}+\frac{3}{2}=1 \longrightarrow a+b=1$

۵)

$f(x)=\begin{cases} 2x-3 & x\geq 1 \\ 2x+3 & x<1 \end{cases}$

$D_f: [-3,+\infty)$

$g(x)=\sqrt{f(x)-x}$

$f(x)-x\geq 0 \longrightarrow f(x)\geq x$

$x\geq 1$

$2x-3-x\geq 0 \longrightarrow x-3\geq 0 \longrightarrow x\geq 3$

$2x+3>x \longrightarrow x+3>0 \longrightarrow x>-3$

$x<1$

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$D_g\ [3,+\infty)\cup(1,-3]$

(6)

$$f(x)=\begin{cases}(a+1)(x+2) & x\geq 1\\ 3a+4x & x<1\end{cases}$$

$$2f(0)=f(-2)+a$$

$$2\big(7(a+1)\big)=3a-4+a$$

$$14a+14=4a-4\rightarrow 10a=-18\rightarrow a=\frac{-18}{10}$$

2

(7) $(2-\sqrt{3})(2+\sqrt{3})=1$

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2=\frac{x+2\sqrt{x}+1}{\sqrt{x}}=\frac{(\sqrt{x}+1)^2}{\sqrt{x}+1-1}$$

عکس را بنویسید

1.5

$$f(x)=\frac{x^2}{x-1}\rightarrow f(2-\sqrt{3})=\frac{7-4\sqrt{3}}{1-\sqrt{3}}$$

$$f(2+\sqrt{3})=\frac{7+4\sqrt{3}}{1+\sqrt{3}}$$

$$\frac{7-4\sqrt{3}}{1-\sqrt{3}}+\frac{7+4\sqrt{3}}{1+\sqrt{3}}=\frac{7-4\sqrt{3}+7\sqrt{3}-12+7+4\sqrt{3}-7\sqrt{3}-12}{-2}$$

$$=\frac{-10}{-2}=5\longrightarrow$$ جواب (5)

$2\sqrt{9}+4$

(8) $2f(x)-3f(-x)=4x^2-x \qquad f(x)=?$

$$3\,\big|\,-3f(x)+2f(-x)=4x^2+x$$

2

$$-5f(x)=20x^2+x\rightarrow f(x)=\frac{20x^2+x}{-5}$$

(9)

$$x=0\rightarrow\quad 2f(0)-0=3m-1$$

1.5

$$x=-2\qquad 0+4f(0)=16+2m+3m-1=5m+15$$

$$9m-3=5m+15\rightarrow 4m=18\rightarrow m=\frac{18}{4}$$

$$f(0)=\frac{25}{4}$$

1-

$$f(x) = ax+b \rightarrow f(x) + f\left(\frac{1}{x}\right) = ax+b + \frac{a}{x} + b$$

$$= \frac{ax^2+bx+a+bx}{x} = \frac{ax^2+2bx+a}{x}$$

$$3x^2-12x+3 = ax^2+2bx+a$$

3

$$a=3,\quad b=-6 \longrightarrow f(x) = 3x-6$$

$$f(-1) = -9$$

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2)

$$x-1 \geq 0 \rightarrow x \geq 1$$

$$3 - \underbrace{\sqrt[4]{x-1}}_{\geq 0} = \sqrt{y+1} \longrightarrow x \leq 82 \quad D_f = [1, 82]$$