تاریخ: / / موضوع:

تکلیف شماره ۴ کمال موسی یازدهم دفتر A

(۱)

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$

$\sqrt{\frac{x^2 - 2x + 1 - x^2}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}}$

$x = \frac{1}{2}$ ← $\frac{-2x+1}{x(x-1)} \geq 0$

$0 \quad 1$

	0		$\frac{1}{2}$		1	
$+$	ن	$-$	0	$+$	ن	$-$

$D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \dfrac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} = \dfrac{\frac{x - 2x - 2}{x(x+1)}}{\frac{3x+6+x-1}{(x-1)(x+2)}} =$

$\dfrac{\frac{-x-2}{x(x+1)}}{\frac{4x+5}{(x-1)(x+2)}}$

$x \neq 0$

$x \neq -1$

$4x \neq -5$

$x \neq \frac{-5}{4}$

$x \neq 1$

$x \neq -2$

$D_f = \mathbb{R} - \{\frac{-5}{4}, -2, -1, 1, 0\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{3}\right)^x - 9\right)\left(32 - 4^x\right)}$ ۲

$$\left(\left(\frac{1}{3}\right)^x - 9\right)\underbrace{\left(32 - 4^x\right)}\geq 0$$

$$2^5 = 2^{2x} \to 2x = 5 \to x = \frac{5}{2}$$

$$\left(\frac{1}{3}\right)^x = 3^2 = \left(\frac{1}{3}\right)^{-2}$$

$$x = -2$$

	-2		$\frac{5}{2}$	
$+$	0	$-$	0	$+$

$$D_f = (-\infty, -2] \cup \left[\frac{5}{2}, +\infty\right)$$

ب) $\sqrt[4]{x-1} + \sqrt{y+1} = 3$

$$\sqrt{y+1} = 3 - \sqrt[4]{x-1}$$

$$x - 1 \geq 0$$

$$x \geq 1 \quad (1)$$

$$3 - \sqrt[4]{x-1} \geq 0$$

$$3 \geq \sqrt[4]{x-1}$$

$$x - 1 \leq 3^4$$

$$x \leq 82 \quad (2) \qquad D_f = [1, 82]$$

$$(1) \cap (2) \Longrightarrow 1 \leq x \leq 82$$

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$$f(x) = \frac{\log_{4}(x^2-x-2)}{\underbrace{\sqrt{x^2-1}+1}} \to + \text{همواره}$$

$$x^2-x-2>0 \to (x-2)(x+1)>0$$

-1		2	
$+$	$-$	$+$	

$\to (-\infty,-1)\cup(2,+\infty)$ ①

$\sqrt{x^2-1}$ ②

$\hookrightarrow x^2-1 \geq 0 \to x^2 \geq 1 \to x\geq 1 \quad x\leq -1 \quad (-\infty,-1]\cup[1,+\infty)$

$$① \cap ② \Longrightarrow D_f = (-\infty,-1)\cup(2,+\infty)$$

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$D_f = [-2,b]$ $\quad \sqrt{3+ax-x^2}$

$a+b=?$

$3-2a-4=0$

$2a=-1 \to a=\frac{-1}{2}$

$$\sqrt{-x^2+ax+3} = \sqrt{(x+2)(-x+\frac{3}{2})} =$$

-2		$b=+\frac{3}{2}$	
$-$	$+$	$-$	

$$\sqrt{-x^2-\frac{1}{2}x+3}$$

$a=\frac{-1}{2} \quad b=\frac{+3}{2} \quad a+b=\frac{3}{2}-\frac{1}{2}=\frac{2}{2}=1$

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$$f(x) = \begin{cases} 3x-2 & , x \geq 1 \\ 2x+3 & , x < 1 \end{cases}$$

$$g(x) = \sqrt{f(x)-x}$$

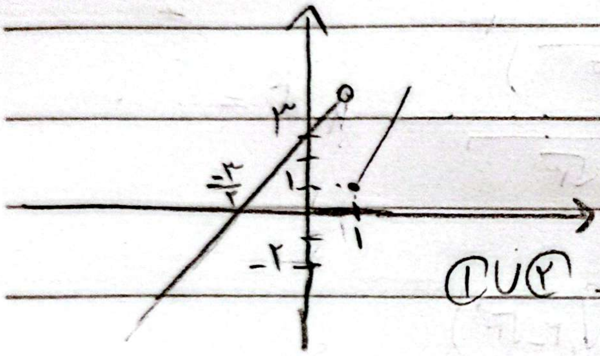
$$f(x)-x \geq 0$$

$$f(x) \geq x \rightarrow 3x-2 \geq x$$

$$2x \geq 2 \rightarrow x \geq 1 \quad (1)$$

$$2x+3 \geq x$$

$$x \geq -3 \quad (2)$$



$$(1) \cup (2) \Rightarrow D_f = [-3, +\infty)$$

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$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases}$$

$$2f(5) = f(-2) + a \longrightarrow 14a+14 = 3a-4+a$$

$$f(5) = (a+1)7 = 7a+7$$

$$10a = -18$$

$$f(-2) = 3a - 4$$

$$a = \boxed{\frac{-18}{10}}$$

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$$f(x)=\sqrt{x}+\frac{۱}{\sqrt{x}}+۲$$

$$f(۲-\sqrt{۳})+f(۲+\sqrt{۳})=$$

$$\sqrt{۲-\sqrt{۳}}+\frac{۱}{\sqrt{۲-\sqrt{۳}}}+۲+\sqrt{۲+\sqrt{۳}}+\frac{۱}{\sqrt{۲+\sqrt{۳}}}+۲=$$

$$\frac{\left(\sqrt{۲-\sqrt{۳}}\right)^۲+۱}{\sqrt{۲-\sqrt{۳}}}+\frac{\left(\sqrt{۲+\sqrt{۳}}\right)^۲+۱}{\sqrt{۲+\sqrt{۳}}}+۴=$$

$$\frac{۲-\sqrt{۳}+۱}{\sqrt{۲-\sqrt{۳}}}+\frac{۲+\sqrt{۳}+۱}{\sqrt{۲+\sqrt{۳}}}+۴=$$

$$\frac{۳-\sqrt{۳}}{\sqrt{۲-\sqrt{۳}}}+\frac{۳+\sqrt{۳}}{\sqrt{۲+\sqrt{۳}}}+۴=$$

$$\frac{(۳-\sqrt{۳})\left(\sqrt{۲+\sqrt{۳}}\right)+(۳+\sqrt{۳})\left(\sqrt{۲-\sqrt{۳}}\right)}{\sqrt{۲-\sqrt{۳}}\times\sqrt{۲+\sqrt{۳}}}+۴=\frac{\sqrt{(۳-\sqrt{۳})^۲\,۲+\sqrt{۳}}+\sqrt{(۳+\sqrt{۳})^۲\,۲-\sqrt{۳}}}{۱}+۴$$

$$=\sqrt{(۳-\sqrt{۳})\,(۳+\sqrt{۳})}+\sqrt{(۳+\sqrt{۳})\,(۳-\sqrt{۳})}+۴=$$

$$\sqrt{۶}+\sqrt{۶}+۴=\boxed{۲\sqrt{۶}+۴}$$

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$$\begin{cases} \left(2f(x) - 3f(-x) = 4x^2 - x\right) \times 2 \\ \left(2f(-x) - 3f(x) = 4x^2 + x\right) \times 3 \end{cases}$$

$$4f(x) - 6f(-x) = 8x^2 - 2x$$
$$6f(-x) - 9f(x) = 12x^2 + 3x$$

$$-5f(x) = 20x^2 + x$$
$$-5f(x) = x(20x+1)$$

$$f(x) = \frac{x(20x+1)}{-5}$$

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$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1$$

$$f(0) = ?$$

$$x=0 \rightarrow 2f(0) = 3m - 1 \longrightarrow$$
$$x=-2 \rightarrow 6f(0) = 4\times4 + 2m + 3m - 1 = 5m + 15$$

$$\begin{cases} 6f(0) = 5m + 15 \\ \left(2f(0) = 3m - 1\right) \times -3 \end{cases}$$

$$5m + 15 - 9m + 3 = 0$$
$$-4m + 18 = 0$$
$$-4m = -18 \rightarrow m = \frac{+18}{4} = \frac{9}{2}$$

$$2f(0) = 3\times\frac{9}{2} - 1 = \frac{27}{2} - 1 = \frac{25}{2} \rightarrow \boxed{f(0) = \frac{25}{4}}$$

موضوع: تاریخ: / /

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$$\underbrace{f(x)}_{ax+b} + f\left(\frac{1}{x}\right) = \frac{3x^2-12x+3}{x}$$

$f(-1)=?$

$$ax+b+\frac{1}{x}a+b = \frac{3x^2-12x+3}{x}$$

$$\frac{x^2+1}{x}a + 2b = \frac{3x^2-12x+3}{x} = 3x-12+\frac{3}{x}$$

$$= \frac{3x^2+3}{x} - 12$$

$$\frac{3x^2+3}{x} = \frac{3(x^2+1)}{x} = a\,\frac{x^2+1}{x} \Longrightarrow a=3$$

$$2b=-12 \rightarrow b=-6$$

$f(x)=3x-6$

$f(-1) = -3-6 = \boxed{-9}$

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