

الف) $F(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)(x-1) - x(x)}{x(x-1)} \rightarrow \frac{x^2 - 1 + 2x - x^2}{x(x-1)} = \frac{1+2x}{x(x-1)}$
 $\rightarrow \frac{-\frac{1}{2} \quad 0 \quad 1}{-\frac{1}{2} \quad -\frac{1}{2} \quad -\frac{1}{2} \quad +}$ $\rightarrow [-\frac{1}{2}, 0) \cup (1, +\infty)$

ب) $F(x) = \frac{1}{x+1} - \frac{2}{x}$ $\xrightarrow{\text{بنام مشترک}} \frac{x+1}{x(x+1)} - \frac{2}{x} = \frac{x+1-2(x+1)}{x(x+1)} = \frac{-x-1}{x(x+1)}$
 $\frac{3}{x-1} + \frac{1}{x+2} \geq 0 \rightarrow \frac{3(x+2) + 1(x-1)}{(x-1)(x+2)} = \frac{3x+6+x-1}{(x-1)(x+2)} = \frac{4x+5}{(x-1)(x+2)}$
 $\rightarrow R - \{-1, 0, -\frac{5}{4}, 2\}$

الف) $F(x) = \sqrt{(\frac{1}{x^2} - 9)(32 - 4x)} \rightarrow \frac{1}{x^2} - 9 \geq 0 \rightarrow \frac{1}{x^2} \geq 9 \rightarrow \frac{1}{x^2} = 9 \rightarrow x = \pm \frac{1}{3}$
 $\rightarrow 32 - 4x \geq 0 \rightarrow -4x \geq -32 \rightarrow x \leq 8 \rightarrow (x^2)^{-1} = \frac{1}{x^2} = 32$
 $\frac{-\frac{1}{3} \quad \frac{1}{3}}{+\frac{1}{3} \quad -\frac{1}{3} \quad +}$ $\rightarrow (-\infty, -\frac{1}{3}] \cup [\frac{1}{3}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} = 2 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$
 $\rightarrow y+1 \geq 0 \rightarrow y \geq -1 \rightarrow [x \geq 1, y \geq -1 \mid (x, y) \in R]$

$f(x) = \log_f(x^2 - x - 2) \rightarrow x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) \rightarrow x = -1, 2$ $\frac{-1 \quad 2}{+\frac{1}{2} \quad -\frac{1}{2} \quad +}$
 $\sqrt{x^2 - 1 + 1} \rightarrow x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x = \pm 1$ $\frac{-1 \quad +1}{+\frac{1}{2} \quad -\frac{1}{2} \quad +}$
 $\rightarrow (-\infty, -1) \cup (2, +\infty)$

$\sqrt{3+4x-x^2} \rightarrow 3+4x-x^2 \geq 0 \rightarrow x^2 - 4x - 3 \leq 0 \rightarrow x^2 - 4x - 3 = 0$
 $\rightarrow x = \frac{4 \pm \sqrt{16+12}}{2} \quad x_1 = -2 \quad x_2 = 6 \quad -2 = \frac{4 - \sqrt{28}}{2}, \quad 6 = \frac{4 + \sqrt{28}}{2}$
 $\rightarrow -2 = \frac{4 - \sqrt{28}}{2} \rightarrow -4 = 4 - \sqrt{28} \rightarrow \sqrt{28} = 8 \rightarrow 28 = 64 \rightarrow 28 = (4+4)^2$
 $\frac{a^2}{a^2} \rightarrow 12 = 16a + 12 \rightarrow 16a = -4 \rightarrow a = -\frac{1}{4} \quad b = \frac{3}{4} \rightarrow -\frac{1}{4} + \frac{3}{4} = 1$

$f(x) = \begin{cases} 3x-2 & : x \geq 1 \\ 2x+3 & : x < 1 \end{cases} \rightarrow g(x) = \sqrt{f(x)-x} \rightarrow f(x)-x \geq 0$
 $\rightarrow 3x-2-x \geq 0 \rightarrow 2x-2 \geq 0 \rightarrow 2x \geq 2 \rightarrow x \geq 1$
 $\rightarrow 2x+3-x \geq 0 \rightarrow x+3 \geq 0 \rightarrow x \geq -3 \rightarrow [-3, +\infty)$

$$f(x) = \begin{cases} (a+1)(x+r) : x > 1 & \rightarrow r f(a) = f(-r) + a \\ ra + rx : x < 1 & \rightarrow r(a+1)(\frac{1}{a+r}) = ra - r + a \end{cases}$$

$$ra + r(\frac{1}{a+r}) = 1fa + rf = fa - f \rightarrow 10a = -11 \rightarrow a = -\frac{11}{10}$$

$$\boxed{a = -1,1}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + r \rightarrow \underbrace{f(r-\sqrt{r})}_a + \underbrace{f(r+\sqrt{r})}_b$$

$$\sqrt{r-\sqrt{r}} + \frac{1}{\sqrt{r-\sqrt{r}}} + r + \sqrt{r+\sqrt{r}} + \frac{1}{\sqrt{r+\sqrt{r}}} + r$$

$$\sqrt{4} + \frac{a+b}{ab} = \sqrt{4} + \sqrt{4} \rightarrow r + r\sqrt{4}$$

$$r f(x) - r f(-x) = rx^2 - x \rightarrow f(x) = ax^2 + bx + c \rightarrow f(-x) = a(-x)^2 + b(-x) + c = ax^2 - bx + c$$

$$\rightarrow r f(x) - r f(-x) = r(ax^2 + bx + c) - r(ax^2 - bx + c)$$

$$(rax^2 + rbx + rc) + (-rax^2 + rbx - rc) = (ra - ra)x^2 + (rb + rb)x + (rc - rc) = 1$$

$$\rightarrow 2ax^2 + 2bx - c \rightarrow ax^2 + bx - c = \frac{1}{2}x^2 - x$$

$$\rightarrow -a = \frac{1}{2} \rightarrow a = -\frac{1}{2}x^2, \quad 2b = -1 \rightarrow b = -\frac{1}{2}x, \quad -c = 0 \rightarrow c = 0$$

$$\rightarrow f(x) = -\frac{1}{2}x^2 - \frac{1}{2}x$$

$$(x+r)f(x) - rx f(x+r) = rx^2 - mx + rm - 1$$

$$f(0) \rightarrow r f(0) = rm - 1 \rightarrow f(0) = \frac{rm-1}{r}$$

$$f(-r) \rightarrow r f(0) = 1q + rm + rm - 1 \rightarrow f(0) = \frac{1d + dm}{r}$$

$$\left. \begin{aligned} \frac{rm-1}{r} &= \frac{1d+dm}{r} \\ \rightarrow 11m-4 &= r \cdot 1 + 1 \cdot m \\ \rightarrow 10m &= 4 \rightarrow m = \frac{2}{5} \end{aligned} \right\}$$

$$\rightarrow f(0) = \frac{r(\frac{2}{5}) - 1}{r} = \frac{\frac{2r}{5} - 1}{r} = \frac{\frac{2d}{5} - 1}{r} = \frac{\frac{2d-5}{5}}{r} = \frac{2d-5}{5r}$$

$$f(x) + f(\frac{1}{x}) = \frac{rx^2 - 1rx + r}{x} \rightarrow f(-1) + f(-1) = \frac{r(-1)^2 - 1r(-1) + r}{-1} = \frac{11}{-1} = -11$$

$$\rightarrow r f(-1) = -11 \rightarrow f(-1) = -\frac{11}{r} \rightarrow \boxed{f(-1) = -9}$$