

(1) $\sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \begin{cases} 1-2x=0 \rightarrow x=\frac{1}{2} \\ x(x-1)=0 \rightarrow x=0, 1 \end{cases}$

$D = (0, \frac{1}{2}] \cup (1, +\infty)$

ب) $y = \frac{\frac{1}{x+1} - \frac{y}{x}}{\frac{y}{x-1} + \frac{1}{x+y}} = \frac{\frac{x-yx-y}{x(x+1)}}{\frac{yx+y+x-1}{(x-1)(x+y)}} = \frac{-x-y}{x(x+1)} \rightarrow \frac{-x-y}{(x-1)(x+y)}$

$\begin{cases} x(x+1) \neq 0 \\ x \neq 0 \\ x \neq -1 \end{cases} \quad \begin{cases} -x+y \neq 0 \\ x \neq \frac{-y}{1} \end{cases} \quad \begin{cases} (x-1) \neq 0 \rightarrow x \neq 1 \\ x \neq -y \end{cases}$

$D = \mathbb{R} - \{-1, 0, -\frac{y}{1}, 1, -y\}$

(2) الف) $f(x) = \sqrt{(\frac{1}{e})^x - 9} (2x - e^x) \geq 0$

$(\frac{1}{e})^x - 9 = 0 \rightarrow (\frac{1}{e})^x = 9 \rightarrow e^{-x} = 9 \rightarrow x = -2$

$2x - e^x = 0 \rightarrow x = \frac{0}{2}$

$D = (-\infty, -2] \cup [\frac{0}{2}, +\infty)$

ب) $\sqrt{x-1} \geq 0 \rightarrow x \geq 1$

$\sqrt{y+1} \geq 0 \rightarrow y \geq -1$

$\sqrt{x-1} \leq 2 \rightarrow x-1 \leq 4 \rightarrow x \leq 5 \rightarrow D = [1, 5]$

(3) $f(x) = \frac{\log_e(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \neq 0$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) = 0 \rightarrow \begin{cases} x = -1 \\ x = 2 \end{cases}$

$x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow |x| > 1 \rightarrow (-\infty, -1) \cup (1, +\infty)$

$D = (-\infty, -1) \cup (2, +\infty)$

(4) $\sqrt{2+ax} - x^2 \geq 0$

$x = -2 \rightarrow 2+a(-2) - (-2)^2 = 2-2a-4 = 0 \rightarrow 2a = -2 \rightarrow a = -1$

$2+ax - x^2 = 0 \rightarrow -x^2 + \frac{1}{2}x + 2 = 0 \rightarrow x^2 - \frac{1}{2}x - 4 = 0$

$x^2 + \frac{1}{2}x - \frac{1}{2} = 0 \rightarrow (x+\frac{1}{2})(x-\frac{1}{2}) = 0 \rightarrow \begin{cases} x = -\frac{1}{2} \\ x = \frac{1}{2} \end{cases} \rightarrow D = [-\frac{1}{2}, \frac{1}{2}]$

$a+b = \frac{1}{2} + (-\frac{1}{2}) = 0$

(5) $f(x) = \begin{cases} 2x-2 & x \geq 1 \\ 2x+2 & x < 1 \end{cases}$

1) $x \geq 1 \rightarrow f(x) = 2x-2 \rightarrow x \geq 1 \rightarrow [1, +\infty)$

2) $x < 1 \rightarrow f(x) = 2x+2 \rightarrow x < -1 \rightarrow (-\infty, -1)$

$D = (-\infty, -1) \cup [1, +\infty)$

$$f(x) = \begin{cases} (a+1)(x+1) & ; x > 1 \\ ra + rx & ; x \leq 1 \end{cases} \quad \begin{aligned} rf(a) &= f(-r) + a \\ f(0) &= v(a+1) \end{aligned} \quad (5)$$

$$f(-r) = ra - r \rightarrow r \times v(a+1) = ra - r + a \Rightarrow 1ra + 1r = ra - r \Rightarrow 1 \cdot a = -1 \Rightarrow a = -\frac{1}{1} = -1$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + r \rightarrow f(r-\sqrt{r}) + f(r+\sqrt{r})$$

$$a = r - \sqrt{r}, b = r + \sqrt{r} = \frac{r+\sqrt{r}}{r-\sqrt{r}} \times \frac{r-\sqrt{r}}{r-\sqrt{r}} = \frac{1}{r-\sqrt{r}} \rightarrow b = \frac{1}{a} \quad (5)$$

$$a \cdot b = (r - \sqrt{r}) \times \left(\frac{1}{r - \sqrt{r}}\right) = 1$$

$$f(r - \sqrt{r}) + f(r + \sqrt{r}) = f(a) + f(b) = \left(\sqrt{a} + \frac{1}{\sqrt{a}} + r\right) + \left(\sqrt{b} + \frac{1}{\sqrt{b}} + r\right) =$$

$$(\sqrt{a} + \sqrt{b}) + \left(\frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}\right) + r \quad \underline{b = \frac{1}{a}} \quad \sqrt{a} + \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{a}} + \sqrt{a} + r =$$

$$r(\sqrt{a} + \frac{1}{\sqrt{a}}) + r \quad \underline{\star}$$

$$\sqrt{a} = \sqrt{r - \sqrt{r}} = \frac{\sqrt{r} - 1}{\sqrt{r}} \times \frac{\sqrt{r}}{\sqrt{r}} = \frac{\sqrt{4} - \sqrt{r}}{r}$$

$$\sqrt{a} + \frac{1}{\sqrt{a}} = \frac{\sqrt{4} - \sqrt{r}}{r} + \frac{\sqrt{4} + \sqrt{r}}{r} = \frac{\sqrt{4}}{r} = \sqrt{4} \rightarrow$$

$$f(r - \sqrt{r}) + f(r + \sqrt{r}) \underline{\star} r\sqrt{4} + r$$

$$rf(x) - rf(-x) = rx^r - x \quad (\star)$$

$$x \rightarrow -x \Rightarrow rf(-x) - rf(x) = rx^r + x$$

$$\begin{cases} rf(x) - rf(-x) = rx^r - x \\ -rf(x) + rf(-x) = rx^r + x \end{cases}$$

$$\underline{-f(x) - f(-x) = rx^r \rightarrow -f(x) - rx^r = f(-x)} \quad \star$$

$$(\star\star) rf(x) - r(-f(x) - rx^r) = rx^r - x \Rightarrow rf(x) + rf(x) + rx^r = rx^r - x$$

$$2f(x) = -rx^r - x \rightarrow f(x) = -\frac{r}{2}x^r - \frac{1}{2}x$$

$$(x+r)f(x) - rx f(x+r) = rx^r - mx + r m - 1$$

$$\underline{x=0} \quad rf(0) = rm - 1 \rightarrow f(0) = \frac{rm-1}{r} \quad (1)$$

$$\underline{x=-r} \quad rf(0) = 1r + rm + rm - 1 \rightarrow rf(0) = 1a + am \rightarrow f(0) = \frac{1a+am}{r} \quad (2)$$

$$(1) = (2) \quad \frac{rm-1}{r} = \frac{1a+am}{r} \rightarrow rm - r = 1a + am \rightarrow rm = 1a + am \rightarrow m = \frac{1a}{r} = \frac{9}{r}$$

$$f(0) = \frac{10}{r}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{5x^2 - 12x + 5}{x}$$

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$$\left. \begin{array}{l} f(x) = ax + b \\ f\left(\frac{1}{x}\right) = \frac{a}{x} + b \end{array} \right\} \rightarrow f(x) + f\left(\frac{1}{x}\right) = ax + b + a\left(\frac{1}{x}\right) + b = \frac{5x^2 - 12x + 5}{x}$$

$$\Rightarrow \frac{ax^2 + bx + a + bx}{x} = \frac{ax^2 + 1b x + a}{x} = \frac{5x^2 - 12x + 5}{x} \Rightarrow a = 5$$

$$2b = -12 \Rightarrow b = -6$$

$$f(x) = 5x - 6 \Rightarrow f(-1) = 5(-1) - 6 = -11$$