

13/1/20

پریامین

-1

$$الف) f(n) = \sqrt{\frac{n-1}{n} - \frac{n}{n-1}} \rightarrow n \neq 0, n \neq 1, \frac{n-1}{n} - \frac{n}{n-1} \geq 0 \rightarrow$$

$$\frac{n^2 + 1 - 2n - n^2}{n(n-1)} \rightarrow \frac{1-2n}{n(n-1)} \geq 0 \quad \begin{array}{c} 0 \quad \frac{1}{2} \quad 1 \\ + \quad | \quad - \quad | \quad + \quad | \quad - \end{array}$$

$$n \in (-\infty, 0) \cup [\frac{1}{2}, 1)$$

5

$$ب) f(n) = \frac{\frac{1}{n+1} - \frac{2}{n}}{\frac{2}{n-1} + \frac{1}{n+2}} \rightarrow n \neq -1, n \neq 0, n \neq 1, n \neq -2,$$

$$\frac{2}{n-1} + \frac{1}{n+2} \neq 0 \rightarrow \frac{2(n+2) + (n-1)}{(n-1)(n+2)} \rightarrow \frac{3n+4+n-1}{n^2+2n-n-2} \rightarrow$$

$$\frac{4n+3}{n^2+n-2} \neq 0 \rightarrow 4n+3 \neq 0 \rightarrow 4n \neq -3 \rightarrow -\frac{3}{4} \neq n$$

$$n \in \mathbb{R} - \left\{ -1, 0, 1, -2, -\frac{3}{4} \right\}$$

پیر یا امینی

-۲

$$f(x) = \sqrt{\underbrace{\left(\left(\frac{1}{x}\right)^x - 9\right)(32 - x^x)}_{\geq 0}}$$

$$\left(\frac{1}{x}\right)^x - 9 = 0 \rightarrow x = -2$$

$$32 - x^x = 0 \rightarrow x = \frac{\omega}{2}$$

$$\begin{array}{c} -2 \qquad \frac{\omega}{2} \\ \hline + \quad - \quad + \end{array}$$

50
1

$$D = (-\infty, -2] \cup \left[\frac{\omega}{2}, +\infty\right)$$

$$\rightarrow \sqrt{x-1} + \sqrt{y+1} = 3 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$$

$$y+1 \geq 0 \rightarrow y \geq -1$$

$$D = \{(x, y) \in \mathbb{R}^2 \mid x \geq 1, y \geq -1\}$$

$$x-1 \geq 0 \rightarrow x \geq 1$$

$$\underbrace{x-1}_{\geq 0} + \underbrace{y+1}_{\geq 0} \rightarrow x \leq 12 - D_f = [1, 12]$$

پیر یا امینی

-۳

$$f(x) = \frac{\log_e (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$$

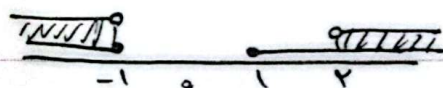
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$$x^2 - x - 2 > 0 \rightarrow \frac{-1 \pm \sqrt{1+8}}{2} \rightarrow (-\infty, -1) \cup (2, +\infty)$$

$$x^2 - 1 \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1 \cup x \leq -1$$

$$\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1$$

هنگامی که عبارت داخل رادیکال منفی باشد



$$(-\infty, -1) \cup (2, +\infty)$$

-۴

$$\sqrt{3 + ax - x^2} \rightarrow D = [-2, b]$$

ریشه‌های دایره عبارتند از

$$3 - 2a - 4 = 0 \rightarrow -1 = 2a \rightarrow a = -\frac{1}{2}$$

صفحات -> کوچکترین مقدار

$$3 - \frac{1}{2}x - x^2 \rightarrow \text{جمع ریشه‌ها} = \frac{\frac{1}{2}}{-1} = -\frac{1}{2}$$

$$b = -\frac{1}{2} - (-2) = \frac{3}{2}$$

$$a + b = -\frac{1}{2} + \frac{3}{2} = 1$$

-۵

$$f(x) - x \geq 0 \begin{cases} x \geq 1 \rightarrow 3x - 2 - x \geq 0 \rightarrow 2x - 2 \geq 0 \rightarrow x \geq 1 \\ x < 1 \rightarrow 2x + 3 - x \geq 0 \rightarrow x + 3 \geq 0 \rightarrow x \geq -3 \end{cases}$$

$$[-3, 1) \cup [1, +\infty) \rightarrow x \geq -3$$

5

پیر یا امینی

(10) - 4

$$f(n) = \begin{cases} (a+1)(n+2) & ; n > 1 \\ 2n+3 & ; n \leq 1 \end{cases}$$

$$2f(a) = f(-2) + a$$

$$a = -1, 1$$

$$f(a) \rightarrow (a+1)(2) = 2a+2$$

$$f(-2) \rightarrow -2 + \cancel{2} = -1$$

$$2(2a+2) = -1 + a \rightarrow 4a+4 = -1+a \rightarrow 3a = -5 \rightarrow a = -\frac{5}{3}$$

-V

$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3}) = \sqrt{4} + 2 + \sqrt{4} + 2 = 2\sqrt{4} + 4$$

$$f(2+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2 = \frac{\sqrt{3}+1}{\sqrt{2}} + \frac{1}{\frac{\sqrt{3}+1}{\sqrt{2}}} + 2$$

$$f(2+\sqrt{3}) = \frac{\sqrt{3}+1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}+1} + 2$$

$$\frac{\sqrt{3}+1}{\sqrt{2}} + \frac{\sqrt{4}-\sqrt{2}}{2} + 2 \rightarrow \frac{\sqrt{4}+\sqrt{2}}{2} + \frac{\sqrt{4}-\sqrt{2}}{2} + 2 \rightarrow$$

$$\frac{\sqrt{4} + \sqrt{2} + \sqrt{4} - \sqrt{2} + 4}{2} = \frac{2\sqrt{4} + 4}{2} = \sqrt{4} + 2$$

$$f(2-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 = \frac{\sqrt{3}-1}{\sqrt{2}} + \frac{1}{\frac{\sqrt{3}-1}{\sqrt{2}}} + 2 =$$

$$\frac{\sqrt{3}-1}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{3}-1} + 2 = \frac{\sqrt{3}-1}{\sqrt{2}} + \frac{\sqrt{4}+\sqrt{2}}{2} + 2 = \frac{\sqrt{4}-\sqrt{2}}{2} + \frac{\sqrt{4}+\sqrt{2}}{2} + 2 =$$

$$\frac{\sqrt{4} - \sqrt{2} + \sqrt{4} + \sqrt{2} + 4}{2} = \frac{2\sqrt{4} + 4}{2} = \sqrt{4} + 2$$

پیرا ایسی
- ۸

$$\textcircled{1} \quad 2f(n) - 2f(-n) = \varepsilon n^2 - n$$

$$2f(-n) - 2f(-(-n)) = \varepsilon(-n)^2 - (-n)$$

$$2f(-n) - 2f(n) = \varepsilon n^2 + n \quad \textcircled{2} \quad -2f(n) + 2f(-n) = \varepsilon n^2 + n$$

حذف $f(-n)$ ←

$$\textcircled{1} \times \textcircled{2} \rightarrow \varepsilon f(n) - 4f(-n) = 12n^2 - 2n \quad \textcircled{3}$$

$$\textcircled{1} \times \textcircled{3} \rightarrow -9f(n) + 4f(-n) = 12n^2 + 2n \quad \textcircled{4}$$

$$-5f(n) = 20n^2 + 2n \quad \div -5 \rightarrow f(n) = \frac{20n^2 + 2n}{-5}$$

$$f(n) = -\left(\frac{20n^2}{5}\right) - \left(\frac{2n}{5}\right) \rightarrow f(n) = -\varepsilon n^2 - \frac{2n}{5}$$

- 9

$$(n+2)f(n) - 2nf(n+2) = \varepsilon n^2 - mn + 2m-1$$

$$f(n) = an^2 + bn + c \rightarrow (an^2 + bn + c)(n+2) - 2n(an^2 + bn + c) = \varepsilon n^2 - mn + 2m-1$$

$$\rightarrow (an^2 + bn + c)(n+2) = \varepsilon n^2 - mn + 2m-1$$

$$an^3 + 2an^2 + bn^2 + 2bn + cn + 2c = \varepsilon n^2 + (2a+b)n^2 + (2b+c)n + 2c$$

$$a(n+2)^2 = a(n^2 + \varepsilon n + \varepsilon) = an^2 + \varepsilon an + \varepsilon a$$

$$b(n+2) = bn + 2b$$

$$2n(an^2 + bn + c) = 2n(an^2 + \varepsilon an + \varepsilon a + bn + 2b + c)$$

$$= 2n(an^2 + (\varepsilon a + b)n + (\varepsilon a + 2b + c)) = 2an^3 + 2(\varepsilon a + b)n^2 + 2(\varepsilon a + 2b + c)n$$

$$= 2an^3 + (2\varepsilon a + 4b)n^2 + (2\varepsilon a + 4b + 2c)n$$

$$(an^3 + (2a+b)n^2 + (2b+c)n + 2c) - (2an^3 + (2\varepsilon a + 4b)n^2 + (2\varepsilon a + 4b + 2c)n) = \varepsilon n^2 - mn + 2m-1$$

پیر یا اسینی

جمع ضرایب x^3 باید صفر شود ←

$$a - 3a - 2a = 0 \rightarrow a = 0$$

$$f(x) = bx + c$$

دوباره دواهل قبل را تکراری کنیم:

$$(bx + c)(x + 2) - 3x(b(x + 2) + c) = \varepsilon x^2 - mx + 3m - 1$$

$$(bx + c)(x + 2) = bx^2 + 2bx + cx + 2c = bx^2 + (2b + c)x + 2c$$

$$3x(b(x + 2) + c) = 3x(bx + 2b + c) = 3bx^2 + 4bx + 3cx =$$

$$3bx^2 + (4b + 3c)x$$

$$\text{معادله اصلی} \rightarrow (bx^2 + (2b + c)x + 2c) - (3bx^2 + (4b + 3c)x) =$$

$$\varepsilon x^2 - mx + 3m - 1$$

ضرایب x^2 ←

$$b - 3b = -2b = \varepsilon \rightarrow b = -\frac{\varepsilon}{2}$$

ضرایب x ←

$$(2b + c) - (4b + 3c) = -m \rightarrow 2b + c - 4b - 3c = -m \rightarrow$$

$$-2b - 2c = -m \xrightarrow{b = -\frac{\varepsilon}{2}} -2(-\frac{\varepsilon}{2}) - 2c = -m \rightarrow \varepsilon - 2c = -m \rightarrow$$

$$m = 2c - \varepsilon$$

ضرایب ثابت ←

$$2c = 3m - 1$$

$$\rightarrow m = 2c - \varepsilon \rightarrow m = 3m - 1 - \varepsilon \rightarrow 2m = 1 - \varepsilon \rightarrow m = \frac{1 - \varepsilon}{2}$$

$$c = \frac{2d}{\varepsilon}$$

9

$$f(x) = -\frac{\varepsilon}{2}x + \frac{2d}{\varepsilon} \rightarrow f(0) = -\frac{\varepsilon}{2}(0) + \frac{2d}{\varepsilon} = \frac{2d}{\varepsilon}$$

پریامی

- 10

$$f(n) = an + b$$

$$f(n) + f\left(\frac{1}{n}\right) = an + b + a\left(\frac{1}{n}\right) + b = an + b + \frac{a}{n} + b = an + \frac{a}{n} + 2b$$

$$f(n) + f\left(\frac{1}{n}\right) = \frac{(3n^2 - 12n + 3)}{n}$$

$$\frac{(3n^2 - 12n + 3)}{n} = \frac{3n^2}{n} - \frac{12n}{n} + \frac{3}{n} = 3n - 12 + \frac{3}{n}$$

$$a = 3 \leftarrow \text{ضریب } n$$

$$a = 3 \leftarrow \frac{1}{n} \text{ ضریب}$$

$$b = -4 \leftarrow 2b = -12 \leftarrow \text{عدد ثابت}$$

$$f(n) = 3n - 4 \rightarrow f(-1) = 3(-1) - 4 = -9$$