١٩,٥

رسا صادقی زاده یازدهم ریاضی ۸

سوال ۱)

$y = \sqrt{x^3 f(x+1)}$ $\qquad f(x) = \left(\frac{1}{2}\right)^{x-2} = f(x) = 2^{-x+2}$

$\xrightarrow{f(x+1)} 2^{-x-1+2} = 2^{-x+1}$

$\Rightarrow y = \sqrt{x^3 \times 2^{-x+1}} \rightarrow x^3 \times 2^{-x+1} \geq 0 \Rightarrow x \geq 0\,,$

$\Rightarrow 2^{-x+1} \geq 0$

$\Rightarrow -x+1 \geq 0\,,\ -x \geq -1$

$x \leq 1$

$\Rightarrow D_f = [0,1]$

سوال ۲)

$f(x) = \sqrt{x + |x+2|} \rightarrow f(-x) = ? \xrightarrow{f(-x)} \sqrt{-x + |2-x|}$

$\Rightarrow -x + |2-x| \geq 0 \xrightarrow{if\ x \geq 0} \cancel{-x - 2 + x} \geq 0 \Rightarrow -2 \geq 0$ ✗

$\xrightarrow{if\ x<0} -x + 2 - x = -2x + 2 \geq 0 \rightarrow x \leq 1 \Rightarrow D_f = (-\infty, 1]$

سوال ۳)

$y = \sqrt{\dfrac{x-1}{f(x)}}$ $\qquad \dfrac{x-1}{f(x)} \geq 0$

	-4		1		2		3
$x-1$	−	−	0	+	+	+	
$f(x)$	+	0	−	−	0	+	0
$\frac{x-1}{f(x)}$	−	∞	+	0	−	∞	+	∞

$D_f = (-4, 1] \cup (2, 3)$

سوال ٤)

$D_{y_1} = D_{y_2} \rightarrow$ مخرج نباید صفر شود $\qquad \dfrac{1}{9x^4 - 7x^2 - 4x - 3}$

$\underbrace{(9x^4 - 6x^2 + 1)}_{(3x^2-1)^2} \underbrace{- x^2 - 4x - 4}_{-(x+2)^2} \rightarrow (3x^2-1)^2 - (x+2)^2 \neq 0$

$\hookrightarrow (3x^2-1)^2 \neq (x+2)^2$

$\Rightarrow 3x^2 - 1 \neq x + 2 \rightarrow 3x^2 - x - 3 \neq 0$

$\hookrightarrow 3x^2 + ax + b \neq 0 \Rightarrow a = -1\,,\ b = -3$

$\sqrt{-(-4)} = 2$ ✓

سوال ۵)

$y=\sqrt{x^4+x-\frac{64}{x^2}-\frac{4}{x}}$, $\sqrt{\frac{x^6+x^4-64-4}{x^2}}$

$\frac{x^6+x^4-64-4x^2}{x^2}\geq 0 \xrightarrow{x^2=t} t^3+t^2-4t-64=t^3-64+t^2-4t$

$=(t-4)(t^2+4t+16)+t(t-4)=(t-4)(t^2+5t+16)$

$x=\pm 2 \Leftarrow x^2=4 \Leftarrow t=4 \Leftarrow \Delta=25-64<0$ ↲

	-2		0		2	
$-$		$+$		$-$		$+$

$\Rightarrow D_f=[-2,0)\cup[2,+\infty)$

۶)

$f(x^3+x)=2x^2-1$ $\qquad f(2)+f(10)=?$

$x^3+x=2 \to x^3+x-2=0 \xrightarrow{\text{تقسیم}}$

$$\begin{array}{l|l} x^3+x-2 & x-1 \\ \hline -x^3+x^2 & x^2+x+2 \\ \quad +x^2+x-2 & \\ \quad -x^2+x & \\ \quad\quad 2x-2 & \\ \quad\quad -2x+2 & \\ \quad\quad\quad 0 & \end{array}$$

$(x-1)(x^2+x+2)=0 \Rightarrow x=1$

$\Delta<0$

$\Rightarrow f(2)=1$

$x^3+x=10 \to \underbrace{x^3+x-8-2}$

$=(x-2)(x^2+4+2x)+(x-2)=(x-2)(x^2+2x+5)$

$\Rightarrow x=2 \Rightarrow f(10)=7$ $\qquad \Delta<0$ ↲

$\Rightarrow f(2)+f(10)=1+7=8$

سوال ۷)

$f(x)=x^3+12x-6x^2-8+8=(x-2)^3+8$

$g(x)=x^3+9x^2+27x+27-27=(x+3)^3-27$

$\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=\frac{(\sqrt[3]{7})^3-27}{(\sqrt[3]{2})^3+8}=\frac{-20}{10}=-2$

هوالله (+)

سوال ۸)

$f(x) = \sqrt{x+4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}+2)^2} = \sqrt{x-4}+2$

$g(x) = \sqrt{x-4\sqrt{x-4}} = \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2|$

$f(x)+g(x) = \sqrt{x-4}+2+|\sqrt{x-4}-2|$

$x \geq 8 \to \sqrt{x-4}+2+\sqrt{x-4}-2 = 2\sqrt{x-4}$ → $a=0$, $b=2$, $c=-4$ → $\frac{a+b}{c} = \frac{-1}{2}$ ✓

$x = 8 \to \sqrt{x-4}+2+0 \to a=2$, $b=-1$, $c=-4$ → $\cancel{\frac{a+b}{c}} = \frac{3}{4}$

مقدار دقیق ندارد

$4 \leq x < 8 \to \sqrt{x-4}+2$

$-\sqrt{x-4}+2 = 4-x$ → $a=-4$, $b=0$, $c \in R$

۲

سوال ۹)

$g(x) = \{(2,1),(3,0),(4,5),(0,4)\}$, $f(x) = \frac{x+2}{x^2-4x+3}$

$\frac{g}{f} =$; $x^2-4x+3 \neq 0$; $x+2 \neq 0 \to x \neq -2$

$(x-3)(x-1) \neq 0 \to x \neq 3,1$

$f(x) = \{(2,-4),(0,\frac{2}{3})\} \to \frac{g}{f} = \{(2,-\frac{1}{4}),(0,6)\}$

۲

سوال ۱۰)

$g(x) = \{(-2,3),(1,-1),(4,2),(-1,0)\}$, $f(x) = \{(1,2),(3,-1),(4,2),(-1,6)\}$

$f(2x) = \{(\frac{1}{2},2),(\frac{3}{2},-1),(2,2),(-\frac{1}{2},6)\}$

$f(x^2) = \{(1,2),(\sqrt{3},-1),(2,2)\}$

$2g^2(x)+1 = \{(-2, 2\times 9+1), (1, 2\times(-1)^2+1), (4, 2\times 2^2+1), (-1, 2\times(0)+1)\} = \{(-2,19),(1,3),(4,9),(-1,1)\}$

۱,۵

۲ $\frac{f}{g} = \{(1,-4);(3,-1)\}$

۱۰ ب) $\begin{cases} x^2 = 1 \rightarrow x = \pm 1 \\ x^2 = 3 \rightarrow x = \pm 3 \\ x^2 = 4 \rightarrow x = \pm 2 \\ x^2 = -1 \rightarrow \times \end{cases} \rightarrow \{(\pm 1, 2)(\pm\sqrt{3}, -1)(\pm 2, 2)\}$