

$$f(n) = \left(\frac{1}{n}\right)^{n-1} - 1 \rightarrow f(n+1) = \left(\frac{1}{n+1}\right)^{n-1} - 1 \rightarrow \left(\frac{1}{n+1}\right)^{n-1} = 1 : n-1=0$$

$$\rightarrow y = \sqrt[n]{f(n+1)} \rightarrow n^{\frac{1}{n}} \times f(n+1) \geq 0$$

$$D_f = [0, 1]$$

$$f(n) = \sqrt{n+|n+1|}$$

$$f(-n) \rightarrow \sqrt{-n+|-n+1|} : \begin{cases} -n+1 \geq 0 \\ -n \geq -1 \\ n \leq 1 \end{cases} \Rightarrow D_f = (-\infty, 1]$$

$$y = \sqrt{\frac{n-1}{f(n)}} \rightarrow D_f = ?$$

$$f(n) \neq 0 \rightarrow n \neq -1, 0, 1$$

$$\frac{n-1}{f(n)} \geq 0 \Rightarrow \begin{cases} n-1 \geq 0 \\ n \geq 1 \end{cases}$$

$$\begin{cases} \frac{1}{n} \in [1, +\infty) \cup (-\infty, -1) \rightarrow (1, 3) \checkmark \\ \frac{-1}{n} \in (-\infty, 1] \cup (-1, 0) \rightarrow (-1, 0) \checkmark \end{cases}$$

$$\Rightarrow D_f = (-1, 0] \cup (1, 3)$$

$$y_1 = \frac{1}{an^2 + bn + c} \stackrel{D_f}{=} y_2 = \frac{1}{n^2 + an + b} \rightarrow \sqrt{-(a \pm b)} = ?$$

$$D_f = \mathbb{R} - \{ \text{roots of } an^2 + bn + c \}$$

$$an^2 + bn + c \neq 0$$

$$4a^2 - 4b^2 + 1 > 0 \Rightarrow 4a^2 - 4b^2 + 1 > 0$$

$$(2a-1)^2 - (2b+1)^2 \neq 0$$

$$(2a-1) \neq (2b+1) \Rightarrow 2a-1 \neq 2b+1 \Rightarrow a \neq b+1$$

$$|2a-1| \neq |2b+1| \Rightarrow 2a-1 \neq 2b+1 \Rightarrow a \neq b+1$$

$$\Rightarrow \sqrt{n^2 - n - 3} = \sqrt{n^2 + (a \pm b)n + b^2}$$

$$\Rightarrow \sqrt{-(-1-3)} = 2$$

$$f(n) = n^2 + n \rightarrow D_f = ? : y = \sqrt{f(n) - f\left(\frac{1}{n}\right)} \geq 0 \rightarrow f(n) \geq f\left(\frac{1}{n}\right)$$

$$\Rightarrow n^2 + n - \frac{1}{n^2} - \frac{1}{n} \geq 0 \rightarrow n^2 + n - \frac{1}{n^2} - \frac{1}{n} \geq 0$$

$$\left(n - \frac{1}{n}\right) \left(n^2 + 1 + \frac{1}{n^2}\right) \geq 0$$

$$n = \frac{1}{n} \Rightarrow n^2 = 1 \Rightarrow n = \pm 1$$

$$\begin{cases} n^2 = 1 \Rightarrow n = \pm 1 \\ n^2 = 1 \Rightarrow n = \pm 1 \end{cases}$$

$$D_f = [-1, 0] \cup [1, +\infty)$$

$$D_f = (-\infty, -1] \cup [1, +\infty)$$

$$f(x^v + x) = x \cdot x^v - 1 \rightarrow f(x) + f(1 \cdot x) = ? \therefore 1 + v = \frac{1}{x}$$

$$n^k, n=2 \rightarrow n=1$$

$$\Rightarrow f(x) = x(1)^x - 1 = 1$$

↳ $f(1.) = n^w + n = 1. \rightarrow n = r$

$$\Rightarrow f(1) = r(r)^{r-1} = r$$

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$$f(n) = n^2 - 4n + 11, \quad g(n) = n^2 + 9n + 41$$

$$= n(n^2 - 4n + 11) \quad = n(n^2 + 9n + 41)$$

$$\rightarrow \frac{g(\sqrt{v}-3)}{f(\sqrt{v}+2)} = ?$$

$$\Rightarrow \frac{(\sqrt{v}-r)(\sqrt{v}-r) + 9(\sqrt{v}-r) + 4v}{(\sqrt{v}+r)(\sqrt{v}+r) - 7(\sqrt{v}+r) + 12} = \frac{(\sqrt{v}-r)(\sqrt{v}+r+9)}{(\sqrt{v}+r)(\sqrt{v}-r-7)} = \frac{\sqrt{v}^2 - r^2}{\sqrt{v}^2 + r^2} = \frac{v-rv}{v+r} = \frac{-r}{1} = -r$$

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$$f(x) = \sqrt{x + \sqrt{x - \varepsilon}}, \quad g(x) = \sqrt{x - \varepsilon \sqrt{x - \varepsilon}}, \quad f(x) + g(x) = a + b\sqrt{x + c} \implies \frac{a+b}{c} = ? \quad x \geq \varepsilon$$

$\Rightarrow \sqrt{n-\epsilon+\epsilon\sqrt{n-\epsilon+\epsilon}} \rightarrow \sqrt{(n-\epsilon+\epsilon)^2} = |n-\epsilon+\epsilon| \rightarrow n-\epsilon+\epsilon \xrightarrow{n \geq \epsilon} n \xrightarrow{g(n) \rightarrow \sqrt{(n-\epsilon+\epsilon)^2} = |n-\epsilon+\epsilon| \xrightarrow{n \geq \epsilon} n}$
 $\Rightarrow f(n) + g(n) = \sqrt{n-\epsilon+\epsilon} + \begin{cases} \sqrt{n-\epsilon-\epsilon} \rightarrow \epsilon\sqrt{n-\epsilon} = \frac{a}{\epsilon} + \frac{b}{\epsilon}\sqrt{n+c-\epsilon} \\ \epsilon - \sqrt{n-\epsilon} \rightarrow \epsilon = \frac{a}{\epsilon} + \frac{b}{\epsilon}\sqrt{n+c-\epsilon} \end{cases}$
 $\Rightarrow \frac{a+b}{\epsilon} \cdot \begin{cases} \frac{a+\epsilon}{-\epsilon} = \frac{1}{\epsilon} \\ \frac{\epsilon+0}{-\epsilon} = -\frac{1}{\epsilon} \end{cases}$

$$f(n) = \frac{n+2}{2^x - 4n + 4} \rightarrow g(n) = \left\{ \left(\frac{1}{2}, 1 \right), (1, 0), \left(\frac{3}{2}, 0 \right), \left(0, \frac{1}{2} \right) \right\} \quad : \quad \frac{g}{y} = \left\{ \left(\frac{1}{2}, -\frac{1}{2} \right), (0, 1) \right\}$$

$$n \neq 1, 2 \quad \rightarrow \quad f(n) = \left\{ \left(\frac{1}{2}, -\frac{1}{2} \right), \left(0, \frac{1}{2} \right) \right\}$$

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$$f(x) = \{(1, 2), (2, -1), (3, 4), (-1, 4)\} \quad / \quad g(x) = \{(-2, 3), (1, -1), (2, 2), (-1, 0)\}$$

الف) $f(x) = \left\{ \left(\frac{1}{x}, x \right), \left(\frac{3}{x}, -1 \right), (x, x), \left(-\frac{1}{x}, x \right) \right\}$

$$\therefore f(x) = \{(1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2)\} \quad (-1, 2)$$

c) $yg^r(n+1) : yg^r(n)^+ = \{(-2, 1), (1, 3), (3, 9), (-1, 1)\}$

$\Rightarrow \frac{2f}{g} \rightarrow \left(\frac{1}{2}, 4 \right), \left(\frac{3}{2}, -2 \right), \left(\frac{5}{2}, 4 \right), \left(\frac{7}{2}, 12 \right)$