

$$f(n) = \left(\frac{1}{n}\right)^{n-1} - 1 \rightarrow f(n+1) = \left(\frac{1}{n+1}\right)^{n-1} - 1 \rightarrow \left(\frac{1}{n+1}\right)^{n-1} = 1 : n-1=0$$

$$\rightarrow y = \sqrt[n]{f(n+1)} \rightarrow n^{\frac{1}{n}} \times f(n+1) \geq 0 \quad n=1$$

$$D_f = [0, 1]$$

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$$f(n) = \sqrt{n+|n+1|}$$

$$f(-n) \rightarrow \sqrt{-n+|-n+1|} : \frac{1}{-n+1} \geq 0 \Rightarrow$$

$$-n+1 \geq 0 \Rightarrow -n \geq -1 \Rightarrow n \leq 1 \rightarrow D_f = (-\infty, 1]$$

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$$y = \sqrt{\frac{n-1}{f(n)}} \rightarrow D_f = ? \rightarrow \frac{n-1}{f(n)} \geq 0 \rightarrow \frac{1}{-n+1} \geq 0$$

$$f(n) \neq 0 \rightarrow n \neq -1, 0, 1$$

$$D_f = \left\{ \frac{1}{2} : [1, +\infty) \cup (-\infty, -1) \cup (0, 1) \rightarrow (0, 1) \checkmark \right. \\ \left. \frac{-1}{2} : (-\infty, 1] \cup (-1, 0) \rightarrow (-1, 0) \checkmark \right.$$

$$\Rightarrow D_f = (-1, 0] \cup (0, 1)$$

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$$y_1 = \frac{1}{an^2 + bn + c} \stackrel{D_f}{=} y_2 = \frac{1}{n^2 + an + b} \rightarrow \sqrt{-a \pm b} = ?$$

$$D_f = \mathbb{R} - \left\{ \frac{-a \pm \sqrt{a^2 - 4b}}{2} \right\}$$

$$an^2 + bn + c \neq 0$$

$$4a^2 - 4b^2 + 1 \Rightarrow n^2 - 4n + 1$$

$$(n^2 - 1)^2 - (n+1)^2 \neq 0$$

$$(n^2 - 1)^2 \neq (n+1)^2 \Rightarrow n^2 - n - 1 = 0 \Rightarrow (1) + 4 \Delta > 0 \checkmark$$

$$|n^2 - 1| \neq |n+1| \Rightarrow n^2 + n + 1 = 0 \Rightarrow \Delta < 0 \checkmark$$

$$\Rightarrow n^2 - n - 1 = n^2 + an + b$$

$$\Rightarrow \sqrt{-(-1-1)} = 2$$

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$$f(n) = n^2 + n \rightarrow D_f = ? : y = \sqrt{f(n) - f\left(\frac{1}{n}\right)} \geq 0 \rightarrow f(n) \geq f\left(\frac{1}{n}\right)$$

$$\Rightarrow n^2 + n - \frac{1}{n^2} - \frac{1}{n} \geq 0 \rightarrow \frac{n^3 - 1}{n^2} + n - \frac{1}{n} \geq 0 \Rightarrow (n - \frac{1}{n}) \left(n^2 + \frac{1}{n^2} + 1 \right) \geq 0$$

$$(n - \frac{1}{n}) \left(n^2 + 1 + \frac{1}{n^2} \right)$$

$$n = \frac{1}{n} \Rightarrow n^2 = 1 \Rightarrow n = \pm 1$$

$$n^2 \geq 1 \Rightarrow n \geq 1 \text{ or } n \leq -1$$

$$D_f = (-\infty, -1] \cup [1, +\infty)$$

$$\frac{1}{n^2} + 1 + \frac{1}{n^2}$$

$$! + 0 \text{ and } \mathbb{R} \leftarrow \Delta < 0 \Rightarrow \Delta > 0$$

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$$f(x^v + x) = x \cdot x^v - 1 \rightarrow f(x) + f(1 \cdot x) = ? \therefore 1 + v = \frac{1}{x}$$

$$x^r, n=r \rightarrow n=1$$

$$\Rightarrow f(x) = x(1)^x - 1 = 1$$

↳ $f(1, \cdot) = 0$ $n^{\text{te}} + n = 1 \rightarrow n = 0$

$$\Rightarrow f(1) = r(r)^{r-1} = r$$

$$f(n) = n^2 - 4n + 12n \quad , \quad g(n) = n^2 + 9n + 4n \quad \rightarrow \quad \frac{g(\sqrt{v}-v)}{f(\sqrt{v}+v)} = ?$$

$$\Rightarrow \frac{(\sqrt{v}-2)(\sqrt{v}-3) + 9(\sqrt{v}-2) + 2v}{(\sqrt{v}+2)(\sqrt{v}+3) - 7(\sqrt{v}+2) + 12} = \frac{(\sqrt{v}-2)(\sqrt{v}+9) + 2v}{(\sqrt{v}+2)(\sqrt{v}-4) + 12} = \frac{\sqrt{v}-2}{\sqrt{v}+2} = \frac{v-4}{v+4} = \frac{-2}{2} = -1$$

$$f(n) = \sqrt{n + \epsilon} \sqrt{n - \epsilon}, \quad g(n) = \sqrt{n - \epsilon} \sqrt{n - \epsilon}, \quad f(n) + g(n) = a + b\sqrt{n + c} \Rightarrow \frac{a+b}{c} = 1, \quad n \geq \epsilon$$

$$\Rightarrow \sqrt{x-\varepsilon} + \varepsilon \sqrt{x-\varepsilon} + \varepsilon \rightarrow \sqrt{(x-\varepsilon+r)^2} = \sqrt{x-\varepsilon+r} \xrightarrow{n \geq \varepsilon} \sqrt{x-\varepsilon} + r \xrightarrow{r \rightarrow \varepsilon} \sqrt{x-\varepsilon} + \varepsilon \Rightarrow g(n) = \sqrt{(x-\varepsilon-r)^2} = \sqrt{x-\varepsilon-r} \xrightarrow{n \geq \varepsilon} \sqrt{x-\varepsilon-r} + r \xrightarrow{r \rightarrow \varepsilon} \sqrt{x-\varepsilon-r} + \varepsilon$$

$$\begin{cases} \sqrt{x-\varepsilon-r} + r & x \geq 1 \\ r - \sqrt{x-r} & \varepsilon \leq x \leq 1 \end{cases}$$

$$\Rightarrow f(n) + g(n) = \sqrt{x-\varepsilon} + \varepsilon + \begin{cases} \sqrt{x-\varepsilon-r} + r \rightarrow r\sqrt{x-\varepsilon} = \frac{a}{\varepsilon} + b\sqrt{x+\varepsilon} - \varepsilon \\ r - \sqrt{x-r} \rightarrow \varepsilon = \frac{a}{\varepsilon} + b\sqrt{x+\varepsilon} - c\varepsilon \end{cases} \quad \frac{a+b}{\varepsilon} \cdot \left\{ \begin{aligned} \frac{a+r}{-\varepsilon} &= -\frac{1}{\sqrt{x-\varepsilon}} \\ \frac{\varepsilon+0}{-\varepsilon} &= -\frac{1}{\sqrt{x-\varepsilon}} \end{aligned} \right.$$

$$f(n) = \frac{n+2}{n^2-n+4} \rightarrow g(n) = \left\{ \left(\frac{0}{1}, 1 \right), (1, 0), (2, 2), \left(\frac{0}{1}, 4 \right) \right\} : \frac{g}{y} = \left\{ \left(1, -\frac{1}{4} \right), (0, 4) \right\}$$

$$n+1, 2 \rightarrow f(n) = \left\{ (1, -\frac{1}{4}), \left(0, \frac{2}{1} \right) \right\}$$

$$f(x) = \{(1, 2), (2, -1), (3, 4), (-1, 4)\} / g(x) = \{(2, 3), (1, -1), (2, 2), (-1, 0)\}$$

الف) $f(x, y) = \left\{ \left(\frac{1}{x}, y \right), \left(\frac{y}{x}, -1 \right), (x, y), \left(-\frac{1}{x}, y \right) \right\}$

→) $f(x) = \{(1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2)\}$

c) $\varphi g^r(n)_{+1} : \varphi g^r(n)^+ = \{(-1, 1), (1, 3), (3, 9), (-1, 1)\}$

$$\Rightarrow \frac{2f}{g} : \left(\frac{1}{2}, 4 \right), \left(\frac{3}{2}, -2 \right), \left(1, 4 \right), \left(-1, 12 \right) : \left\{ (1, -4), (3, -1), \left(-1, \frac{1}{2} \right) \right\} = \left\{ (1, -4), (3, -1) \right\}$$