

① $f(x) = \frac{1}{x-2} - 1 \rightarrow f(x+1) = f\left(\frac{1}{x}\right) - 1$

$\underbrace{x^2}_{\leq 0} \underbrace{(2^{1-x} - 1)}_{\geq 0} \geq 0$
 $\underbrace{2^{1-x}}_{\geq 1} - 1 \geq 0 \rightarrow 2^{1-x} \geq 1 \rightarrow 1-x \leq 0 \rightarrow x \leq 1$

$D_f = \mathbb{R} - \{0, 1\}$

$D_f = [1, 1]$

$\frac{1}{x-2}$

② $f(x) = \sqrt{x + |x+2|} \rightarrow f(-x) = \sqrt{|1-x| - x}$
 شرط: $|1-x| - x \geq 0$

$|1-x| - x \geq 0 \rightarrow 1-x \geq x \rightarrow 1 \geq 2x \rightarrow x \leq \frac{1}{2}$

$x-2 \geq x \rightarrow 2 \leq 0$ (غیرممکن)
 $D_f = (-\infty, \frac{1}{2}]$

③ $y = \sqrt{\frac{x-1}{f(x)}}$
 شرط: $\frac{x-1}{f(x)} \geq 0$
 نقاط بحرانی: $-4, 2, 3$

$\frac{x-1}{f(x)} \geq 0 \rightarrow D_f = (-\infty, 1] \cup (2, 3)$

④ $9x^2 - 4x^2 - x^2 - 4x - 4 + 1 \rightarrow$

$\frac{-x^2 - 4x - 4}{-(x+2)^2} = \frac{(9x^2 - 4x^2 + 1)}{(3x^2 - 1)^2} \sqrt{-(a+b)} = \sqrt{f} \geq \frac{1}{2}$

$(3x^2 - 1)^2 - (x+2)^2 \neq 0 \rightarrow x+2 \leq 3x^2 - 1$

$3x^2 - x \leq 3 \rightarrow 3x^2 - x - 3 \leq 0$

$3x^2 - x - 3 \leq 3x^2 + ax + b \rightarrow b \leq -3, a \leq -1$

$$f(x) \leq x^2 + x \quad y = \sqrt{f(x) - f\left(\frac{f}{2}\right)}$$

(2)

$$f(x) - f\left(\frac{f}{2}\right) \geq 0 \rightarrow f(x) \geq f\left(\frac{f}{2}\right)$$

$$x^2 + x \geq \frac{f^2}{x^2} + \frac{f}{2} \rightarrow x^2 - \frac{f^2}{x^2} \leq x - \frac{f}{2}$$

1, 2, 3

$$\left(x - \frac{f}{2}\right) \left(x^2 + \frac{14}{x^2} + x\right) + x - \frac{f}{2} \geq 0$$

1, 2, 3

$$x^2 + \frac{14}{x^2} + x - \frac{f}{2} \geq 0 \rightarrow \frac{(x^4 + 14 + \omega x^2)(x - \frac{f}{2})}{x^2} \geq 0$$

رابطه زیر را از دسترس خارج می‌کنیم

با این فرض

$$-1 \quad 0 \quad +1 \quad D_f = (-\infty, -1] \cup [1, +\infty)$$

$$[-1, 0) \cup (0, 1]$$

$$f(x^2 + x) = 1 - x^2 \rightarrow f(1) + f(1) \leq x^2 + x \leq 1$$

$$x \leq 1 \rightarrow 1 - x^2 \leq 1 \leq 1$$

$$x^2 + x \leq 1 \rightarrow x \leq 1 \rightarrow 1 - x^2 \leq 1 \leq 1$$

$$f(x) \leq x^2 - 4x^2 + 12x$$

(3)

$$f(\sqrt[3]{x} + 1) \leq (1 + 1 + 4\sqrt[3]{x} + 12\sqrt[3]{x}) - 4(\sqrt[3]{x} + 1\sqrt[3]{x} + 8)$$

$$+ 12(\sqrt[3]{x} + 1) \leq 10$$

$$g(x) = x^3 + 9x^2 + 12x$$

$$g(\sqrt[3]{v} - 1) \leq (v - 1 - 9\sqrt[3]{v} + 12\sqrt[3]{v}) + 9(\sqrt[3]{v} - 1 - 4\sqrt[3]{v} + 12\sqrt[3]{v})$$

$$+ 12(\sqrt[3]{v} - 1) \leq v - 1 \leq -1$$

$$\frac{g(\sqrt[3]{v} - 1)}{f(\sqrt[3]{v} + 1)} \leq \frac{-1}{1} \leq -1$$

$$f(x) \leq \sqrt{(x-\varepsilon) + \varepsilon \sqrt{x-\varepsilon}} + \varepsilon$$

(1)

$$f(x) \leq \sqrt{(\sqrt{x-\varepsilon} + r)^2} \rightarrow f(x) \leq |\sqrt{x-\varepsilon} + r|$$

$$g(x) \leq \sqrt{(x-\varepsilon) - \varepsilon \sqrt{x-\varepsilon}} + \varepsilon \rightarrow g(x) \leq |\sqrt{x-\varepsilon} - r|$$

$$f(x) + g(x) \leq a + b\sqrt{x+c} \xrightarrow{[r, 1]} \sqrt{x-\varepsilon} + r + r$$

$$-\sqrt{x-\varepsilon} \leq \varepsilon \xrightarrow{x \geq 1} \sqrt{x+\varepsilon} + r + \sqrt{x-\varepsilon} - r \leq 0 + rx$$

$$\sqrt{x-\varepsilon} \rightarrow 0 + r\sqrt{x-\varepsilon} \geq a + b\sqrt{x+c}$$

$$\frac{a+b}{c} \leq \frac{0+r}{-\varepsilon} \leq \frac{-1}{r}$$

$$x^r - \varepsilon x + r + 0 \rightarrow (x-r)(x-1) \neq 0$$

(9)

$$\frac{-r}{x+r}$$

$$\frac{x^r - \varepsilon x + r}{1+r} \quad f(r) = -\varepsilon \quad f(0) = \frac{r}{r}$$

$$\frac{g}{f} = \left\{ \left(r, \frac{1}{\varepsilon} \right), (0, r) \right\}$$

$$f(rx) \leq \left\{ \left(\frac{1}{r}, r \right), (1/r, -1), (r, r), \left(\frac{-1}{r}, r \right) \right\}$$

(10)

$$f(x^r) \leq \left\{ (1, r), (\sqrt{r}, -1), (-\sqrt{r}, -1), (r, r) \right\}$$

1, r, 0

$$r g^r(x) + 1 \leq \left\{ (-r, 1/r), (1, r), (r, r), (-1, 1) \right\}$$

$$\frac{rf}{g} \leq \left\{ (1, -\varepsilon), (r, -1) \right\}$$

$$\begin{aligned}
 \text{ب ۱۰) } \left\{ \begin{array}{l} a^r = 1 \rightarrow a = \pm 1 \\ a^r = r \rightarrow a = \pm r \\ a^r = r \rightarrow a = \pm r \\ a^r = -1 \rightarrow X \end{array} \right. &\rightarrow \left\{ (\pm 1, r) (\pm \sqrt{r}, -1) (\pm r, r) \right\}
 \end{aligned}$$