Year: یازدهم Month: تکالیف Dey: ۵ ( ) Subject: آزاد بی‌نژاد

۱

$f(x) = \left(\frac{1}{2}\right)^{x-2} - 1 \longrightarrow f(x+1) = f\left(\frac{1}{2}\right)^{x-1} - 1$

$\underbrace{x^3}_{\leq 0}\ \underbrace{(2^{1-x} - 1)}_{0} \geq 0 \qquad 2^{1-x} - 1 \leq 0 \longrightarrow 2^{1-x} \leq 1 \qquad \underbrace{1 - x \leq 0} \qquad x \leq 1$

$D_f = R - \{0, 1\}$

۲

$f(x) = \sqrt{x + |x+2|} \xrightarrow[\text{قرینه}]{} f(-x) = \sqrt{|2-x| - x}$

$|2-x| - x \geq 0 \longrightarrow 2 - x \geq x \longrightarrow 1 \geq x$

$x - 2 \geq x \longrightarrow 2 \leq 0 \longrightarrow$ غیرممکن

$fD = (-\infty, +1]$

۳

$y = \sqrt{\frac{x-1}{\underbrace{f(x)}}}^{\ \frac{1}{2}}$ با توجه به نمودار: ۴-، ۲ و ۳

-4		1		2		3	
$-$	+		$-$		+		$-$

$\frac{x-1}{f(x)} \geq 0 \longrightarrow D_f = (-4, +1] \cup (2, 3)$

۴

$9x^4 - 6x^2 - x^2 - 4x - 4 + 1 \longrightarrow$

$\underbrace{-x^2 - 4x - 4}_{-(x+2)^2}\ \underbrace{(9x^4 - 6x^2 + 1)}_{(3x^2-1)^2} \qquad \sqrt{-(a+b)} = \sqrt{4} = 2$

$(3x^2 - 1)^2 - (x+2)^2 \neq 0 \longrightarrow x + 2 = 3x^2 - 1$

$3x^2 - x = 3 \longrightarrow 3x^2 - x - 3 = 0$

$3x^2 - x - 3 = 3x^2 + ax + b \longrightarrow b = -3, a = -1$

Year: ______ Month: ______ Dey : ______ ( )   **Subject:** ______

(5)

$f(x) = x^3 + x \qquad y = \sqrt{f(x) - f(\frac{4}{x})}$

$f(x) - f(\frac{4}{x}) \geqslant 0 \longrightarrow f(x) \geqslant f(\frac{4}{x})$

$x^3 + x \geqslant \frac{64}{x^3} + \frac{4}{x} \longrightarrow x^3 - \frac{64}{x^3} \leqslant x - \frac{4}{x}$

$(x - \frac{4}{x})(x^2 + \frac{16}{x^2} + 4) + x - \frac{4}{x} \geqslant 0$ $\qquad 2, -2$

$x^2 + \frac{16}{x^2} + \overset{5}{4+1} \geqslant 0 \longrightarrow \frac{(x^4 + 16 + 5x^2)(x - \frac{4}{x})}{x^2} \geqslant 0$ همیشه بزرگتر از صفر

نباید صفر بشه

	-2		0		$+2$	
$-$	0	$+$		$-$	0	$+$

$D_f = (-\infty, -2] \cup [2, +\infty)$

(6)

$f(x^3 + x) = 2x^2 - 1 \longrightarrow f(2) + f(10) = \longrightarrow x^3 + x = 2$

$x = 1 \longrightarrow 2 \times 1 - 1 = 1$ $\qquad f(2) + f(10) = 8$

$x^3 + x = 10 \longrightarrow x = 2 \longrightarrow 2 \times 4 - 1 = 7$ $\qquad 1 \quad 7$

(7)

$f(x) = x^3 - 6x^2 + 12x$

$f(\sqrt[3]{2} + 2) = (2 + 8 + 6\sqrt[3]{4} + 12\sqrt[3]{2}) - 6(\sqrt[3]{4} + 4\sqrt[3]{2} + 4)$

$+ 12(\sqrt[3]{2} + 2) = 10 \qquad g(x) = x^3 + 9x^2 + 27x$

$g(\sqrt[3]{7} - 3) = (7 - 27 - 9\sqrt[3]{49} + 27\sqrt[3]{7}) + 9(\sqrt[3]{49} - 6\sqrt[3]{7} + 9)$

$+ 27(\sqrt[3]{7} - 3) = 7 - 27 = -20$

$\frac{g(\sqrt[3]{7} - 3)}{f(\sqrt[3]{2} + 2)} = \frac{-20}{10} = -2$

8

$$f(x) = \sqrt{(x-4) + 4\sqrt{x-4} + 4}$$

$$f(x) = \sqrt{\left(\sqrt{x-4}+2\right)^2} \longrightarrow f(x) = \left|\sqrt{x-4}+2\right|$$

$$g(x) = \sqrt{(x-4) - 4\sqrt{x-4} + 4} \longrightarrow g(x) = \left|\sqrt{x-4}-2\right|$$

$$f(x)+g(x) = a + b\sqrt{x+c} \xrightarrow{[4, 8]} \sqrt{x-4}+2+2$$

$$-\sqrt{x-4} = 4 \xrightarrow{x \geq 8} \sqrt{x-4}+2+\sqrt{x-4}-2 = 0+2\sqrt{x-4} \longrightarrow 0+2\sqrt{x-4} = a+b\sqrt{x+c} \longrightarrow$$

$$a = 0,\quad b = 2,\quad c = -4$$

$$\frac{a+b}{c} = \frac{0+2}{-4} = \frac{-1}{2}$$

9

$$x^2 - 4x + 3 \neq 0 \longrightarrow \underset{3}{(x-3)}\,\underset{1}{(x-1)} \neq 0$$

$$-2 \longleftarrow \frac{x+2}{\underbrace{x^2-4x+3}_{1,\ 3}} \qquad f(2) = -4 \qquad f(0) = \frac{2}{3}$$

$$\frac{g}{f} = \left\{\left(2, \frac{-1}{4}\right), (0, 6)\right\}$$

10

$$f(2x) = \left\{\left(\frac{1}{2}, 2\right), (1.5, -1), (2, 2), \left(\frac{-1}{2}, 6\right)\right\}$$

$$f(x^2) = \left\{(1, 2)\ (\sqrt{3}, -1)\ (-\sqrt{3}, -1)\ (2, 2)\right\}$$

$$3g^2(x)+1 = \left\{(-2, 19), (1, 13), (3, 9), (-1, 1)\right\}$$

$$\frac{2f}{g} = \left\{(1, -4), (3, -1)\right\}$$