

$$f(x+1) = \frac{1}{x^2} - 1 = \frac{1}{x^2} - 1$$

$$x^2 f(x+1) \geq 0 \Rightarrow \begin{array}{c|ccc} & 0 & 1 & \\ \hline x^2 & - & + & + \\ \hline f(x+1) & + & + & - \\ \hline & - & + & - \end{array} \quad D = [0, 1]$$

$\frac{1}{x^2} = 1 \Rightarrow x = 1$

$$f(-x) = \sqrt{-x + |2-x|} \rightarrow D_{f(x)} = -x + |2-x| \geq 0$$

$$\begin{cases} -x + 2 - x \geq 0 \rightarrow -2x \geq -2 \rightarrow 2x \leq 2 \rightarrow x \leq 1 \\ -x + x - 2 \geq 0 \text{ نمی شود} \end{cases} \Rightarrow D = (-\infty, 1]$$

$$\frac{x-1}{f(x)} \geq 0$$

$$\begin{array}{c|ccc} & -f & 1 & 2 \\ \hline x-1 & - & - & + \\ \hline f(x) & + & - & - \\ \hline y & - & + & - \end{array} \Rightarrow D = (-f, 1] \cup (2, +\infty)$$

$$4x^2 - \sqrt{x^2 - 4x - 3} \neq 0 \rightarrow (x^2 - x - 3)(\sqrt{x^2 - x - 3}) \neq 0 \Rightarrow x^2 - x - 3 \neq 0 \rightarrow D_{y_1} = \mathbb{R} - \left\{ \frac{1 \pm \sqrt{13}}{2} \right\}$$

$$x^2 + ax + b \neq 0 \rightarrow D_{y_2} = D_{y_1} \rightarrow x_{y_2} = \frac{-a \pm \sqrt{a^2 - 4b}}{2} = \frac{1 \pm \sqrt{13}}{2} \Rightarrow -a = 1 \Rightarrow a = -1$$

$$a^2 - 4b = 13 \quad a = -1 \quad 1 - 4b = 13 \Rightarrow b = -3$$

$$\sqrt{-(a+b)} = \sqrt{-(-1-3)} = \sqrt{4} = 2, -2$$

$$f(x) - f\left(\frac{1}{x}\right) \geq 0 \rightarrow f(x) \geq f\left(\frac{1}{x}\right) \rightarrow x^3 + x \geq \frac{1}{x^3} + \frac{1}{x}$$

$$\begin{array}{c|ccc} & -1 & 0 & 1 \\ \hline & - & + & - \end{array} \Rightarrow (-1, 0] \cup [1, +\infty)$$

$$x^r + x = r \rightarrow x = 1 \quad f(r) = r - 1 = 1$$

$$x^r + x = 1 \rightarrow x = r \rightarrow f(1) = r(1) - 1 = r$$

$$f(r) + f(1) = 1 + r = \boxed{1}$$

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$$f(x) = x^r - 4x^r + 12x = (x - r)^r + 1 \quad \frac{g(\sqrt{r} - r)}{f(\sqrt{r} + r)} = \frac{(\sqrt{r} - r)^r - r}{(\sqrt{r} + r)^r + 1}$$

$$g(x) = x^r + 4x^r + 12x = (x + r)^r - r$$

$$\Rightarrow \frac{(\sqrt{r})^r - r}{(\sqrt{r})^r + 1} = \frac{\sqrt{r} - r}{r + 1} = \frac{-r}{1} = \boxed{-r}$$

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$$(f(x) + g(x))^r = (a + b\sqrt{x+c})^r \rightarrow \sqrt{x+c} = \frac{a+b\sqrt{x+c}}{r} \rightarrow \sqrt{x+c} = a + b\sqrt{x+c}$$

$$x \geq r \rightarrow rx + rx - 19 = rx - 19 = f(x - r) \rightarrow \sqrt{x - r} = a + b\sqrt{x+c}$$

$$\rightarrow r\sqrt{x - r} = a + b\sqrt{x+c} \rightarrow \begin{matrix} b = r \\ a = 0 \\ c = -r \end{matrix}$$

$$\frac{a+b}{c} = \frac{0+r}{-r} = \boxed{-\frac{1}{r}}$$

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$$g(x) = \{(r, 1)(1, 0)(r, 2)(0, r)\} \quad f(x) = \{(r, -r)(1, \frac{r}{r})(0, \frac{r}{r})(r, \frac{r}{r})\}$$

$$\frac{g}{f} = \{(r, -\frac{1}{r})(0, r)\}$$

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$$f(x) = \{(\frac{r}{r}, r)(\frac{r}{r}, -1)(r, r)(\frac{1}{r}, r)\}$$

$$f(x) = \{(1, r)(-1, r)(\sqrt{r}, -1)(-\sqrt{r}, -1)(r, r)(-r, r)\}$$

$$rg(x) + 1 = \{(-r, 19)(1, 3)(19, 4)(-1, 1)\}$$

$$\frac{rf}{g} = \{(1, -r)(r, -1)\}$$

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