

$$f(x) = \frac{1}{x} - 1 \rightarrow f(x+1) = \frac{1}{x+1} - 1 \rightarrow f(x+1) = \frac{1}{x} - 1 \rightarrow \begin{array}{c} | \\ + \\ | \\ - \end{array}$$

$$y = \sqrt{x^3 f(x+1)}$$

$$x^3 f(x+1) \geq 0 \rightarrow \begin{array}{c} 0 \\ | \\ - \\ | \\ + \\ | \\ - \end{array} \rightarrow D_f = [0, 1]$$

$$f(-x) = \sqrt{-x + |-x+2|}$$

$$-x + |-x+2| \geq 0 \rightarrow x \leq |-x+2| \xrightarrow{(\cdot)^2} x^2 \leq x^2 - 4x + 4 \rightarrow 4x \leq 4 \rightarrow x \leq 1$$

$$D_f = (-\infty, 1]$$

$$y = \sqrt{\frac{x-1}{f(x)}} \quad \frac{x-1}{f(x)} \geq 0 \quad \begin{array}{c} x-1 \xrightarrow{x=1} \\ f(x) \\ x = -4, 2, 3 \end{array}$$

$$\begin{array}{c} -4 \quad -2 \quad 1 \quad 2 \quad 3 \\ | \quad | \quad | \quad | \quad | \\ - \quad + \quad - \quad + \quad + \end{array}$$

$$D_f = (-4, 1] \cup (2, 3)$$

$$y_1 = \frac{1}{9x^2 - 7x^2 - 4x - 3} \rightarrow 9x^2 - 7x^2 - 4x - 3 \neq 0 \rightarrow 9x^2 - 4x - 3 \neq 0$$

$$(3x^2 - 1)^2 - (x+2)^2 \neq 0 \rightarrow (3x^2 - 1)^2 = (x+2)^2 \xrightarrow{\sqrt{\quad}} |3x^2 - 1| = |x+2|$$

$$\textcircled{1} 3x^2 - 1 = x+2 \rightarrow 3x^2 - x - 3 = 0$$

$$\textcircled{2} 3x^2 - 1 = -x-2 \rightarrow 3x^2 + x + 1 = 0 \rightarrow \Delta < 0 \Rightarrow D_{y_1} = D_{y_2} \rightarrow D_{y_1} = \mathbb{R} - \left\{ \begin{array}{l} \text{ریشه های معادله} \\ 3x^2 - x - 3 = 0 \\ 3x^2 + x + 1 = 0 \end{array} \right\}$$

$$3x^2 - x - 3 = 3x^2 + ax + b \rightarrow \boxed{a = -1}, \boxed{b = -3} \quad \sqrt{-(-1-3)} = 2$$

$$f\left(\frac{x}{2}\right) = \left(\frac{x}{2}\right)^3 + \frac{x}{2} \quad y = \sqrt{f(x) - f\left(\frac{x}{2}\right)} \quad f(x) - f\left(\frac{x}{2}\right) \geq 0 \rightarrow x^3 + x - \left(\frac{x}{2}\right)^3 - \frac{x}{2} \geq 0$$

$$x^3 - \left(\frac{x}{2}\right)^3 + x - \frac{x}{2} \geq 0 \rightarrow \left(x - \frac{x}{2}\right)\left(x^2 + x + \frac{14}{2x}\right) + \left(x - \frac{x}{2}\right) \geq 0 \rightarrow \left(x - \frac{x}{2}\right)\left(x^2 + x + \frac{14}{2x}\right) \geq 0$$

$$\rightarrow x - \frac{x}{2} \geq 0 \rightarrow \begin{array}{c} -2 \quad 2 \\ | \quad | \\ + \quad + \end{array} \rightarrow D_f = (-\infty, -2] \cup [2, +\infty)$$

$\Delta < 0 \Rightarrow a > 0$
همواره مثبت

$x = \pm 2, x \neq 0$

$f(1) \rightarrow x^3 + x = 1 \rightarrow x^3 + x - 1 = 0 \rightarrow (x-2)(x^2 + 2x + 4) + (x-2) = 0$ $(x-2)(x^2 + 2x + 4) = 0 \rightarrow x-2 = 0 \rightarrow x=2 \rightarrow f(1) = 2(2)^2 - 1 = 7$ $f(2) \rightarrow x^3 + x = 2 \rightarrow x^3 + x - 2 = 0 \rightarrow (x-1)(x^2 + x + 2) = 0 \rightarrow x-1 = 0 \rightarrow x=1$ $f(2) = 2(1)^2 - 1 = 1$ $f(2) + f(1) = 7 + 1 = 8$	6
$f(x) = x^3 - 4x^2 + 12x + 8 - 8 \rightarrow (x-2)^3 + 8 \rightarrow f(\sqrt[3]{2} + 2) = (\sqrt[3]{2} + 2)^3 + 8 = 1$ $g(x) = x^3 + 9x^2 + 27x + 27 - 27 \rightarrow (x+3)^3 - 27 \rightarrow g(\sqrt[3]{2} - 3) = (\sqrt[3]{2} - 3)^3 - 27 = -2$ $\frac{g(\sqrt[3]{2} - 3)}{f(\sqrt[3]{2} + 2)} = \frac{-2}{1} = -2$	7
$f(x) + g(x) = \sqrt{x+4}\sqrt{x-4} + \sqrt{x-4}\sqrt{x-4} = \sqrt{x-4+4}\sqrt{x-4+4} + \sqrt{x-4-4}\sqrt{x-4-4}$ $= \sqrt{x-4+4}^2 + \sqrt{x-4-4}^2 = \sqrt{x-4+4} + \sqrt{x-4-4} \rightarrow \begin{cases} \sqrt{x-4+4} + \sqrt{x-4-4} & x \geq 4 \\ \sqrt{x-4+4} - \sqrt{x-4-4} & x < 4 \end{cases}$ $\textcircled{1} \rightarrow 0 + 2\sqrt{x-4} \rightarrow a=0 \quad b=2 \quad c=-4$ $\textcircled{2} \rightarrow 4 + 0\sqrt{x-4} \rightarrow a=4 \quad b=0 \quad c=-4$ $\frac{a+b}{c} \rightarrow \frac{0+2}{-4} = -\frac{1}{2}$ $\frac{4+0}{-4} = -1$	
$f(x) = \frac{x+2}{x^2-4x+3} \rightarrow D_f = \mathbb{R} - \{1, 3\} \quad D_g = \{2, 0, 3, 1\}$ $D_f \cap D_g = \{0, 2\}$ $\frac{g}{f} \rightarrow f(x) \neq 0 \rightarrow x \neq -2 \Rightarrow \frac{g}{f} = \left\{ (2, \frac{1}{2}), (0, 4) \right\}$ $f(2) = \frac{4}{-5} \quad g(2) = 1 \rightarrow \frac{1}{-\frac{4}{5}} \rightarrow -\frac{5}{4}$ $f(0) = \frac{2}{3} \quad g(0) = 4 \rightarrow \frac{4}{\frac{2}{3}} = 6$	9
$f(2x) = \left\{ (\frac{1}{2}, 2), (\frac{3}{2}, -1), (2, 2), (\frac{5}{2}, 4) \right\}$ باید در این‌ها اول را مساوی 2x قرار دهیم و در این صورت بدست آوریم مثلاً $2x=1 \rightarrow x=\frac{1}{2}$	
$f(x^2) = \left\{ (1, 2), (2, 2), (3, -1), (4, -1), (5, 2), (6, 2) \right\}$ در این‌ها اول را مساوی x^2 قرار دهیم و در این صورت بدست آوریم مثلاً $x^2=1 \rightarrow x=\pm 1$ $2g^2(x) + 1 = \left\{ (-2, 19), (1, 3), (3, 9), (1, -1) \right\}$ در این‌ها دوم را به توان 2 رسانده و دوباره یکی کنیم و باید جمع می‌کنیم. $D_f \cap D_g = \{1, 3, -1\} \quad \frac{2f}{g} = \left\{ (1, -4), (3, -1), (-1, \frac{1}{2}) \right\} = \left\{ (1, -4), (3, -1) \right\}$	10