تلفیق ۵ — سایه‌ها دیانا... (۱۴٫۲۵)

۱.

$\sqrt{x^3 f(x+1)} \Rightarrow x^3 f(x+1) \geq 0$

$f(x+1) = \left(\frac{1}{2}\right)^{(x+1-2)} - 1$

$x^3 \left(\left(\frac{1}{2}\right)^{x-1} + 1\right) \geq 0$

x	x^3	$f(x+1)$	$\left(\frac{1}{2}\right)^{x-1}+1$
			صفر
$x<0$	منفی	$x<1$ مثبت	منفی
$x=1$	مثبت	0	0
$x>1$	مثبت	منفی	منفی
$x=0$	صفر		

$D = [0, 1]$

(۳)

۲.

$D_{f(x)}:\ x + |x+2| \geq 0 \rightarrow x + x + 2 \geq 0 \rightarrow 2x \geq -2 \quad x \geq -1$

$x - x - 2 \geq 0$ ضعیف

$D_f = [-1, +\infty)$ قرینه

$D_{f(-x)} = (-\infty, 1]$

(۲)

۳.

$\frac{x-1}{f(x)} \geq 0$ → 1

$f(x)$ → $2, 3, -2$

	-2	1	2	3	
$-$		$+$	$-$	$+$	$-$

$D = (-2, 1] \cup (2, 3)$

(۲)

۴.

$\frac{1}{(3x^2-1)^2 - (x+2)^2} = \frac{1}{(3x^2-1+x+2)(3x^2-1-x-2)}$

اتحاد مزدوج

$3x^2+x+1 \quad 3x^2-x-3$

هیچ‌گاه صفر نمی‌شود چون دلتا منفی

همان مخرج g_2

$\sqrt{-(-1+3)} = \sqrt{-2}$

$\sqrt{-(-1-3)} = 2$

$g_2 = \frac{1}{3x^2 + ax + b}$

$a = -1, \ b = -3$

(۱٫۷۵)

۵-

$f(x) - f\left(\frac{4}{x}\right) = x^2 - x - \frac{64}{x^2} + \frac{4}{x} \geq 0$

$y = x^2$

$\frac{x^6 - x^5 + 4x^2 - 64}{x^3} \geq 0 \Rightarrow \frac{y^3 - y^2 + 4y - 64}{x^3} \geq 0$

$4 \Rightarrow x^2 = 4$

$x = \pm 2$

	-2	0	2	
[illegible]	$+$	$-$	$+$	

$D = [-2, 0) \cup [2, +\infty)$

(۲)

۶-

$x^3 + x = 10 \Rightarrow x = 2$ (توسط آزمایش) $\quad F(10) = 2x^2 - 1 = 2(2)^2 - 1 = 7$

$x^3 + x = 2 \Rightarrow$ (توسط آزمایش) $x = 1 \quad f(2) = 2(1)^2 - 1 = 1$

(۲)

$F(10) + F(2) = 1 + 7 = 8$

۷-

$f(x) = x^3 - 6x^2 + 12x = (x-2)^3 + 2^3 \Rightarrow f(\sqrt[3]{2}+2) = (\sqrt[3]{2}+2-2)^3 + 2^3 = 2 + 8 = 10$

$g(x) = x^3 + 9x^2 + 27x = (x+3)^3 - 3^3 \Rightarrow g(\sqrt[3]{7}-3) = (\sqrt[3]{7}-3+3)^3 - 3^3 = 7 - 27 = -20$

$\Rightarrow \dfrac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)} = \dfrac{-20}{10} = \boxed{-2}$

(۲)

۸-

$\sqrt{x + 4\sqrt{x-4}} + \sqrt{x - 4\sqrt{x-4}} = \sqrt{(x-4) + 4\sqrt{x-4} + 4} + \sqrt{(x-4) - 4\sqrt{x-4} + 4}$

$= \sqrt{(\sqrt{x-4}+2)^2} + \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}+2| + |\sqrt{x-4}-2| = \sqrt{x-4} + 2 + \sqrt{x-4} - 2$

$= 0 + 2\sqrt{x-4} \Rightarrow a = 0 \quad b = 2 \quad c = -4 \quad x \geq 4$

(۲)

$\dfrac{a+b}{c} = \dfrac{0+2}{-4} = -\dfrac{1}{2}$

۹-

$F(2) = \dfrac{2+2}{4-8+3} = \dfrac{4}{-1} = -4 \qquad F(1) = \dfrac{1+2}{1-4+3} =$ تن

$F(3) = \dfrac{3+2}{9-12+3} = \dfrac{5}{0} =$ تن $\qquad F(0) = \dfrac{0+2}{3} = \dfrac{2}{3}$

(۲)

$\dfrac{g}{f}: \dfrac{g(2) = 1}{f(2) = -4} = \boxed{-\dfrac{1}{4}} \qquad \dfrac{g}{f}: \dfrac{g(0)}{f(0)} = \dfrac{4}{\frac{2}{3}} = \boxed{6}$

$= \{(2, -\tfrac{1}{4}), (0, 6)\}$

۱۰-

الف) $2x = 1 \Rightarrow x = \frac{1}{2}$, $2x = 3 \Rightarrow x = \frac{3}{2}$, $2x = 4 \Rightarrow x = 2$, $2x = -1 \Rightarrow x = -\frac{1}{2}$

$F(2x) = \{(\tfrac{1}{2}, 2), (\tfrac{3}{2}, -1), (2, 2), (-\tfrac{1}{2}, 5)\}$

ب) $x^2 = 1 \Rightarrow 1 = x$, $3 = x^2 \Rightarrow x = \sqrt{3}$, $4 = x^2 \Rightarrow x = \pm 2$, $-1 = x^2 \notin$

$F(x^2) = \{(1, 2), (\pm\sqrt{3}, -1), (\pm 2, 2)\}$

ج) $g(-2) = 2(9) + 1 = 19 \quad g(1) = 2 + 1 = 3 \quad g(-1) = 3$

$2g^2(x) + 1 = \{(-2, 19), (-1, 3), (3, 9), (-1, 3)\}$

(۱٫۵)

د) $\dfrac{2f}{g} = \{(3, -1), (1, 4)\}$

$\dfrac{2 \times 2}{-1} = -4$