ملیکا محمد حسینی

۱۹٫۵

(۱)

$f(x+1) = \left(\frac{1}{2}\right)^{(x+1)-2} - 1 = \left(\frac{1}{2}\right)^{x-1} - 1$

$x^3\left(\left(\frac{1}{2}\right)^{x-1} - 1\right) \geq 0$

if $x<1 \to +$, $x=1 \to 0$, $x>1 \to -$

چون $x^3 \geq 0$ پس $x \geq 0$ ✓ **۲**

$D = [0,1]$ ۲ (+ هردو) $0<x<1$ ✓ ۱ $x=1$ (عبارت دوم $=0$) ✓

(۲)

$f(-x) = \sqrt{-x + |-x+2|} = \sqrt{-x+|2-x|}$

$\xrightarrow{x\leq 2} |2-x| = 2-x \to -x+(2-x) = 2-2x \to 2-2x \geq 0 \to 2 \geq 2x \to x \leq 1$ ✓ ; $-x+|2-x| \geq 0$ **۳**

$\xrightarrow{x>2} |2-x| = x-2 \to -x+(x-2) = -2$ چون $-2<0$ پس غ ق ق

$D = (-\infty, 1]$

(۳)

$\sqrt{x-1} \geq 0 \to x-1 \geq 0 \to x \geq 1$, $f(x) \neq 0$

$(-4,0] \cup [0,2) \cup (2,3)$ **۱٫۵**

-4 ، 2 ، 3 جزء جواب های ما نمی باشند

(۴)

$D_{y_1, y_2} = \mathbb{R} - \{$ریشه های مخرج$\}$

$9x^4 - 7x^2 - 4x - 3 = (3x^2 - x - 3)(3x^2 + x + 1)$; $\Delta > 0$, $\Delta < 0$

$3x^2 + ax + b = 3x^2 - x - 3 \to a=-1 \quad b=-3$; $a+b = -1-3 = -4$; $\sqrt{-(a+b)} = \sqrt{4} = 2$ **۲**

(۵)

$f\left(\frac{4}{x}\right) = \left(\frac{4}{x}\right)^3 + \frac{4}{x} = \frac{64}{x^3} + \frac{4}{x} \to y = \sqrt{x^3 + x - \left(\frac{64}{x^3} + \frac{4}{x}\right)}$

$y = \sqrt{x^3 + x - \frac{64}{x^3} - \frac{4}{x}} \to x^3 + x - \frac{64}{x^3} - \frac{4}{x} \geq 0$, $x \neq 0$

$\frac{x^6 + x^4 - 64 - 4x^2}{x^3} \geq 0$, $x \neq 0$ $\xrightarrow{x^2 = t} t^3 + t^2 - 4t - 64 = (t-4)(t^2+5t+16)$ **۲**

$(x^2-4)(x^4+5x^2+16)$

$x^2 - 4 = 0 \to x^2 = 4 \to x = \pm 2$

	-2		0		2	
$-$		$+$		$-$		$+$

$D = [-2, 0) \cup [2, +\infty)$

(۶)

$x^3 + x = 2 \xrightarrow{x=1} 2(1)^2 - 1 = 1$

$x^3 + x = 10 \xrightarrow{x=2} 2(2)^2 - 1 = 7$ $\longrightarrow 7 + 1 = \boxed{8}$ **۲**

(۷)

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$x^3 + 9x^2 + 27x \longrightarrow (x+3)^3 - 27$

$g(\sqrt[3]{7} - 3) = ((\sqrt[3]{7} - 3) + 3)^3 - 27 = (\sqrt[3]{7})^3 - 27 = 7 - 27 = \boxed{-20}$ **۲**

$x^3 - 6x^2 + 12x \longrightarrow (x-2)^3 + 8$ $\longrightarrow \frac{-20}{10} = \boxed{-2}$ ✓

$f(\sqrt[3]{2} + 2) = ((\sqrt[3]{2} + 2) - 2)^3 + 8 = (\sqrt[3]{2})^3 + 8 = 2 + 8 = \boxed{10}$

حلیه محمد حسینی

$A^2 = f^2 + g^2 + 2fg \longrightarrow (x+6\sqrt{x-5}) + (x-6\sqrt{x-5}) + 2\sqrt{(x+6\sqrt{x-5})(x-6\sqrt{x-5})}$

$A^2 = x^2 + 2\sqrt{x^2 - (6\sqrt{x-5})^2} = 2x + 2\sqrt{x^2 - 144(x-5)}$ (۱)

$x^2 - 144x + 720 \rightsquigarrow (x-8)^2 \rightsquigarrow \sqrt{(x-8)^2} = |x-8|$

اگر $5 \le x \le 8$ ⟵ $|x-8| = 8-x$

$A^2 = 2x + 2(-x+8) = 16 \longrightarrow A = 4$ (۲)

اگر $x \ge 8$ ⟵ $|x-8| = x-8$

$A^2 = 2x + 2(x-8) = 4x - 16 = 4(x-4) \longrightarrow A = 2\sqrt{x-4}$

$f(x) + g(x) = a + b\sqrt{x+c} \longrightarrow a=0 \quad b=2 \quad c=-4$

$\frac{0+2}{-4} = \frac{2}{-4} = -\frac{1}{2}$

(۹)

$f(x) = \frac{x+2}{x^2 - 4x + 3} = \frac{x+2}{(x-3)(x-1)} \longrightarrow x \neq 1, 3$ تعریف نشده

$x=2 \longrightarrow \frac{4}{(-1)(1)} = -4 \longrightarrow \frac{g}{f}(2) = \frac{1}{-4} = -\frac{1}{4}$ (۴)

$x=0 \longrightarrow \frac{2}{(-3)(-1)} = \frac{2}{3} \longrightarrow \frac{g}{f}(0) = 6$

$\frac{g}{f} = \left\{\left(2, -\frac{1}{4}\right), (0, 6)\right\}$

(۱۰)

الف) $\left\{\left(\frac{1}{2}, 2\right), \left(\frac{5}{2}, 1\right), (2, 2), \left(-\frac{1}{2}, 4\right)\right\}$

ب) $\{(1,2), (-1,2), (\sqrt{5}, -1), (-\sqrt{5}, 1), (2,2), (-2,2)\}$ (۳)

ج) $2(g(x))^2 + 1 = \{(-2, 19), (1, 3), (2, 9), (-1, 1)\}$

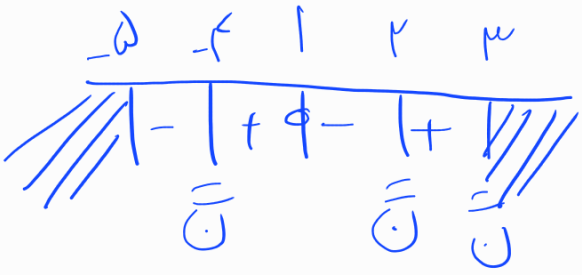
د) $D_h \longrightarrow \{1, 2, -1, \frac{1}{2}\}$ ✓

$\frac{2 \times f(1)}{g(1)} = -4$

$\frac{2 \times f(2)}{g(2)} = -1$

$\longrightarrow \{(1, -4), (2, -1)\}$

۳) ریشه $f(n) = -۴, ۲, ۳$ و ریشه صورت هم $n=۱$



$D_f = (-۴, ۱] \cup (۲, ۳)$