ملیکا محمد حسینی

(1)

$f(x+1) = \left(\frac{1}{2}\right)^{(x+1)-2} - 1 = \left(\frac{1}{2}\right)^{x-1} - 1$

$x^3\left(\left(\frac{1}{2}\right)^{x-1} - 1\right) \geq 0$

$\text{if } \begin{cases} x<1 \to + \\ x=1 \to 0 \\ x>1 \to - \end{cases}$

چون $x^3 \geq 0$ پس $x \geq 0$ ✓

$D = [0,1]$

۲ (+ هردو) $0<x<1$ ✓

۲ $x=1$ (عبارت دوم $=0$) ✓

(2)

$f(-x) = \sqrt{-x + |-x+2|} = \sqrt{-x+|2-x|}$

$\xrightarrow{x\leq 2} |2-x| = 2-x \to -x+(2-x) = 2-2x \rightsquigarrow -x+|2-x| \geq 0$

$\xrightarrow{x>2} |2-x| = x-2 \to -x+(x-2) = -2 \rightsquigarrow$

$2-2x \geq 0 \to 2 \geq 2x \to x \leq 1$ ✓

چون $-2<0$ پس غ ق ق

$D = (-\infty, 1]$

(3)

$\sqrt{x-1} \geq 0 \to x-1 \geq 0 \to x \geq 1 \quad , \quad f(x) \neq 0$

$(-4,0] \cup [0,2) \cup (2,3)$

-4، 2، 3 جزء جواب های ما نمی باشند

(4)

$D_{y_1, y_2} = \mathbb{R} - \{\text{ریشه‌های مخرج}\}$

$9x^4 - 7x^2 - 4x - 3 = \underbrace{(3x^2 - x - 3)}_{\Delta>0}\underbrace{(3x^2+x+1)}_{\Delta<0}$

$3x^2+ax+b = 3x^2-x-3 \to a=-1 \quad b=-3 \rightsquigarrow a+b = -1-3 = -4$

$\sqrt{-(a+b)} = \sqrt{4} = 2$

(5)

$f\left(\frac{4}{x}\right) = \left(\frac{4}{x}\right)^3 + \frac{4}{x} = \frac{64}{x^3} + \frac{4}{x} \to y = \sqrt{x^3+x-\left(\frac{64}{x^3}+\frac{4}{x}\right)}$

$y = \sqrt{x^3+x-\frac{64}{x^3}-\frac{4}{x}} \to x^3+x-\frac{64}{x^3}-\frac{4}{x} \geq 0 \quad , \quad x \neq 0$

$\frac{x^6+x^4-64-4x^2}{x^3} \geq 0 \quad , \quad x\neq 0 \quad \xrightarrow{x^2=t} t^3+t^2-4t-64 = (t-4)(t^2+5t+16)$

$(x^2-4)(x^4+5x^2+16)$

$x^2-4=0 \to x^2=4 \to x=\pm 2$

	-2		0		2	
		$-$	$+$	$-$	$+$	

$D = [-2,0) \cup [2,+\infty)$

(6)

$x^3+x=2 \xrightarrow{x=1} 2(1)^2-1 = 1 \longrightarrow 7+1 = \boxed{8}$

$x^3+x=10 \xrightarrow{x=2} 2(2)^2-1 = 7$

(7)

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$x^3+9x^2+27x \longrightarrow (x+3)^3-27$

$g(\sqrt[3]{7}-3) = \left((\sqrt[3]{7}-3)+3\right)^3 - 27 = (\sqrt[3]{7})^3 - 27 = 7-27 = \boxed{-20}$

$x^3-6x^2+12x \longrightarrow (x-2)^3+8$

$f(\sqrt[3]{2}+2) = \left((\sqrt[3]{2}+2)-2\right)^3+8 = (\sqrt[3]{2})^3+8 = 2+8 = \boxed{10}$

$\longrightarrow \frac{-20}{10} = \boxed{-2}$ ✓

ملکه محمد حسینی

$A^2 = f^2 + g^2 + 2fg \longrightarrow (x+4\sqrt{x-4}) + (x-4\sqrt{x-4}) + 2\sqrt{(x+4\sqrt{x-4})(x-4\sqrt{x-4})}$ (8)

$A^2 = x^2 + 2\sqrt{x^2-(4\sqrt{x-4})^2} = 2x + 2\sqrt{\underbrace{x^2-16(x-4)}}$

$x^2-16x+64 \rightsquigarrow (x-8)^2 \rightsquigarrow \sqrt{(x-8)^2} = |x-8|$

اگر $4 \le x \le 8 \longleftarrow |x-8| = 8-x$

$A^2 = 2x+2(-x+8) = 16 \longrightarrow A = 4$

اگر $x \ge 8 \longleftarrow |x-8| = x-8$

$A^2 = 2x+2(x-8) = 4x-16 = 4(x-4) \longrightarrow A = 2\sqrt{x-4} \longrightarrow a=0 \quad b=2 \quad c=-4$

$f(x)+g(x) = a+b\sqrt{x+c}$

$\boxed{\frac{0+2}{-4} = \frac{2}{-4} = -\frac{1}{2}}$

$f(x) = \frac{x+2}{x^2-4x+3} = \frac{x+2}{(x-3)(x-1)} \longrightarrow x \neq 1, 3$ مقدار ندارد (9)

$x=2 \longrightarrow \frac{4}{(-1)(1)} = -4 \longrightarrow \frac{g}{f}(2) = \frac{1}{-4} = -\frac{1}{4}$

$x=0 \longrightarrow \frac{2}{(-3)(-1)} = \frac{2}{3} \longrightarrow \frac{g}{f}(0) = 6$

$\frac{g}{f} = \left\{\left(2, -\frac{1}{4}\right), (0, 6)\right\}$

الف) $\left\{\left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, 1\right), (2, 2), \left(-\frac{1}{2}, 6\right)\right\}$ (10)

ب) $\{(1,2), (-1,2), (\sqrt{3},-1), (-\sqrt{3},1), (2,2), (-2,2)\}$

ج) $\underset{2\times(3)^2+1}{2(g(x))^2+1} = \{(-2,19), (1,3), (3,9), (-1,1)\}$

د) $D_f \longrightarrow \{\underbrace{1, 3}_{\checkmark}, \cancel{-1}\} \longrightarrow \frac{2\times f(1)}{g(1)} = -4$

$\frac{2\times f(3)}{g(3)} = -1 \longrightarrow \{(1,-4), (3,-1)\}$