آلا کرمانی ۱۹,۷۵ پاسخنامه تشریحی شماره ۵ یازدهم ریاضی A

① $f(x)=(\frac{1}{2})^{x-2}-1 \,,\; y=\sqrt{x^3 f(x+1)} \Rightarrow \underbrace{x^3}_{x\geq 0}\big(\underbrace{(\frac{1}{2})^{x-1}-1}_{x\leq 1}\big)\geq 0 \Rightarrow$

$\begin{array}{ccccc} & 0 & & 1 & \\ - & \phi & + & \phi & - \end{array}$ $\quad D_f=[0,1]$

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② $f(x)=\sqrt{x+|x+2|} \,,\; f(-x)=\sqrt{-x+|2-x|} \Rightarrow |2-x|-x\geq 0 \Rightarrow |2-x|\geq x$

$\left.\begin{cases} 2-x\geq x \Rightarrow 2\geq 2x \Rightarrow 1\geq x \\ 2-x\leq -x \Rightarrow -2\leq 0 \end{cases}\right\} \Rightarrow D_f=(-\infty,1]$

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③ $y=\sqrt{\frac{x-1}{f(x)}} \Rightarrow \frac{x-1 \longrightarrow x=1}{f(x) \longrightarrow f(x)\neq 0 \Rightarrow x=\{-4,2,3\}}\geq 0$

$\begin{array}{ccccccccc} & -4 & & 1 & & 2 & & 3 & \\ - & \phi & + & \phi & - & \phi & + & \phi & \end{array}$

$D_f=(-4,1]\cup(2,3)$

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④ $y_1=\frac{1}{9x^4-7x^2-4x-3\neq 0} \,,\; y_2=\frac{1}{3x^2+ax+b\neq 0} \quad \sqrt{-(a+b)}=?$

$(3x^2-x-3)\underbrace{(3x^2+x+1)}_{\Delta<0 = \text{ریشه ندارد}}$

ریشه‌ها $=\frac{1-\sqrt{37}}{6} \,,\; \frac{1+\sqrt{37}}{6}$

$3x^2-x-3=3x^2+ax+b \Rightarrow a=-1 \,,\; b=-3$

$\sqrt{-(-1-3)}=2$

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⑤ $f(x)=x^3+x \,,\; y=\sqrt{f(x)-f(\frac{4}{x})} \Rightarrow x^3+x-(\frac{64}{x^3})-(\frac{4}{x})\geq 0 \Longrightarrow$

$\frac{x^6+x^4-64-4x^2}{x^3}\geq 0 \Rightarrow \frac{(x^2-4)(x^4+5x^2+16)}{x^3}\geq 0$ $\quad \Delta<0 =$ همیشه مثبت $\quad (x^2-4)\neq 0 \Rightarrow x\neq \pm 2$

$x^3\neq 0 \Rightarrow x\neq 0$

$\begin{array}{ccccccc} & -2 & & 0 & & 2 & \\ - & \phi & + & \phi & - & \phi & + \end{array} \Rightarrow D_f=[-2,0)\cup[2,+\infty)$

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⑥ $f(x^3+x)=2x^2-1 \,,\; f(2)+f(10)=? \Rightarrow f(2) \Rightarrow x^3+x=2 \Rightarrow x(x^2+1)=2 \Rightarrow x=1$

$f(2)=2(1)^2-1=1$

$f(10) \Rightarrow x^3+x=10 \Rightarrow x(x^2+1)=10 \Rightarrow x=2 \Rightarrow f(10)=2(2)^2-1=7$

$f(2)+f(10)=7+1=8$

۷)

$f(x)=x^3-6x^2+12x$ ، $g(x)=x^3+9x^2+27x$ ، $\dfrac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=?$

$f(\sqrt[3]{2}+2)=(\sqrt[3]{2}+2)^3-6(\sqrt[3]{2}+2)^2+12(\sqrt[3]{2}+2)=2+\cancel{6\sqrt[3]{4}}+\cancel{12\sqrt[3]{2}}$
$+8-6(\cancel{\sqrt[3]{4}}+\cancel{4\sqrt[3]{2}}+\cancel{4})+12\sqrt[3]{2}+24=10$

$g(\sqrt[3]{7}-3)=(\sqrt[3]{7}-3)^3+9(\sqrt[3]{7}-3)^2+27(\sqrt[3]{7}-3)=7-9\sqrt[3]{49}+$
$81\sqrt[3]{7}-27+9(\sqrt[3]{49}-6\sqrt[3]{7}+9)+27\sqrt[3]{7}-81=-20$

$$\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=\frac{-20}{10}=-2$$

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۸)

$f(x)+g(x)=\underbrace{a+b\sqrt{x+c}}\Rightarrow t=\sqrt{x-4}\Rightarrow t\geq 0\ ,\ x=t^2+4$

$f(x)=\sqrt{t^2+4+4t}\ \overset{x\geq 4}{\Rightarrow}\ \sqrt{(t+2)^2}=t+2\ ,\ g(x)=\sqrt{t^2+4-4t}\Rightarrow\sqrt{(t-2)^2}=|t-2|$

$f(x)+g(x)=\begin{cases}t+2+t-2=2t\\ t+2-t+2=4\end{cases}$

$\dfrac{a+b}{c}=\dfrac{2}{-4}=-\dfrac{1}{2}$

$f(x)+g(x)=2t=2\sqrt{x-4}=a+b\sqrt{x+c}\Rightarrow a+b=2\ ,\ c=-4$

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۹)

$f(x)=\dfrac{x+2}{x^2-4x+3}$ ، $g(x)=\{(2,1),\cancel{(1,0)},\cancel{(3,5)},(0,4)\}$

$(x-1)(x-3)$

$\dfrac{g}{f}\Rightarrow x=2,0\Rightarrow f(0)=\dfrac{0+2}{0+0+3}=\dfrac{2}{3}\ ,\ f(2)=\dfrac{4}{4-8+3}=-4$

$\dfrac{g}{f}=\{(2,\dfrac{-1}{4}),(0,6)\}$

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۱۰)

الف) $f(2x)=\{(\frac{1}{2},2),(\frac{3}{2},-1),(2,2),(\frac{-1}{2},9)\}$

ب) $f(x^2)=\{(\pm 1,2),(\pm\sqrt{3},-1),(\pm\sqrt{4},2)\}$

ج) $2g^2(x)+1=\{(-2,19),(1,3),(3,9),(-1,1)\}$

د) $\dfrac{2f}{g}=\{(1,-4)(3,-1)\}$

۱٫۷۵