۱-

$f(x)=\left(\frac{1}{2}\right)^{x-2}-1 \rightsquigarrow y=\sqrt{x^2 f(x+1)}=?$

$x^2 f(x+1)\geq 0 \to f(x+1)=\left(\frac{1}{2}\right)^{(x+1)-2}-1=\left(\frac{1}{2}\right)^{x-1}-1 \begin{cases} \text{مثبت} & x<1 \\ \text{صفر} & x=1 \\ \text{منفی} & x>1 \end{cases}$

بازه‌های مجاز $\Longrightarrow x=0 \ ، \ 0<x<1 \ ، \ x=1$

بازه‌های غیرمجاز $\Longrightarrow x>1$

$D=[0,1]$

۲-

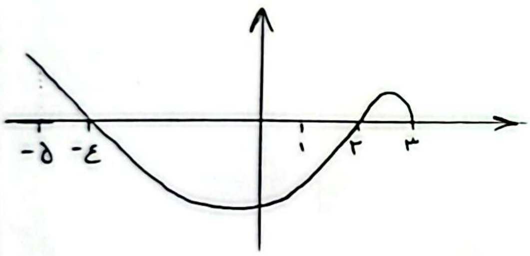
$f(x)=\sqrt{x+|x+2|} \rightsquigarrow f(-x)=?$

$f(-x)=\sqrt{-x+|-x+2|}=\sqrt{-x+|x-2|} \rightsquigarrow -x+|x-2|\geq 0 \Leftrightarrow |x-2|\geq x$

$x\geq 2 \to |x-2|=x-2 \rightsquigarrow$ غیرممکن

$x<2 \to |x-2|=2-x \to 2-x\geq x \to 2\geq 2x \to x\leq 1$ $\quad D=(-\infty,1]$

۳-



$y=\sqrt{\frac{x-1}{f(x)}}=?$

$f(x)=0$ if $x=3$ یا $x=2$ یا $x=-4 \Rightarrow \begin{cases} x<-4 & f(x)>0 \\ x>3 & f(x)<0 \\ -4<x<2 & f(x)<0 \\ 2<x<3 & f(x)>0 \end{cases}$

$\sqrt{\frac{x-1}{f(x)}} \to f(x)\neq 0$ و $\frac{x-1}{f(x)}\geq 0$

بازه‌های غیرمجاز $\Rightarrow (-\infty,-4) \ ، \ x=-4 \ ، \ (1,2) \ ، \ x=2 \ ، \ x=3 \ ، \ x>3$

بازه‌های مجاز $\Rightarrow (-4,1] \ ، \ (2,3)$

$D=(-4,1]\cup(2,3)$

۴-

$y_1=\frac{1}{9x^4-7x^3-4x-3}$ و $y_2=\frac{1}{3x^2+ax+b} \to \sqrt{-(a+b)}=?$

$\to (3x^2+ax+b)(3x^2+cx+d)$

$\to 9x^4+(3c+3a)x^3+(ac+3b+3d)x^2+(ad+bc)x+bd \to bd=-3$

$\to a=-c$

با عددگذاری می‌توان فرض کرد $\longrightarrow a=1$ و $b=1 \longrightarrow a+b=2 \quad \sqrt{-(2)}=\sqrt{-2}$ غم

اکبر مارانی

۵-

$f(x) = x^3 + x \qquad y = \sqrt{f(x) - f\left(\frac{4}{x}\right)} = ?$

$\hookrightarrow x^3 + x - \left(\frac{64}{x^3} + \frac{4}{x}\right) = \frac{x^6 + x^4 - 4x^2 - 64}{x^3}$

$x \neq 0 ,\ \frac{N(x)}{x^3} \geq 0 \Rightarrow x^2 = t \rightsquigarrow t^3 + t^2 - 4t - 64$

$\Rightarrow N(x) = (x^2 - 4)(x^4 + 5x^2 + 16) \longrightarrow x^2 \geq 4 \rightarrow N(x) \geq 0 ,\ x > 0$

$\downarrow$

$|x| \geq 2$

$\hookrightarrow x^2 \leq 4 \rightarrow N(x) \leq 0 ,\ x < 0 \Rightarrow$ غیرق ق

$-2 \leq x < 0 ,\ x = 0$

$D = [-2, 0) \cup [2, \infty)$

۶-

$f(x^3 + x) = 2x^2 - 1 \qquad f(2) + f(10) = ?$

$x^3 + x = 10 \rightarrow x^3 + x - 10 = 0 \rightarrow x = 2 \qquad x^3 + x = 2 \rightarrow x^3 + x - 2 = 0 \rightarrow x = 1$

$f(10) = 2 \times 2^2 - 1 = 7 \qquad f(2) = 2 \times 1^2 - 1 = 1$

$f(2) + f(10) = 7 + 1 = 8$

۷-

$f(x) = x^3 - 6x^2 + 12x ,\ g(x) = x^3 + 9x^2 + 27x \qquad \frac{g(\sqrt[3]{7} - 3)}{f(\sqrt[3]{2} + 2)} = ?$

$\sqrt[3]{7} - 3 = a \rightarrow a^3 = 7 - 9\sqrt[3]{49} + 27\sqrt[3]{7} - 27 \longrightarrow g(\sqrt[3]{7} - 3) = -27\sqrt[3]{7} + 88$

$\sqrt[3]{2} + 2 = b \rightarrow b^3 = (\sqrt[3]{2} + 2)^3 = 2 + 6\sqrt[3]{4} + 12\sqrt[3]{2} + 8 = 10 + 12\sqrt[3]{2} + 6\sqrt[3]{4}$

$\hookrightarrow f(\sqrt[3]{2} + 2) = -12\sqrt[3]{2} - 2 = -2(6\sqrt[3]{2} + 1)$

$\hookrightarrow \frac{27\sqrt[3]{7} - 88}{2(6\sqrt[3]{2} + 1)}$

۸-

$f(x) = \sqrt{x + 4\sqrt{x - 4}} ,\ g(x) = \sqrt{x - 4\sqrt{x - 4}}$

$x \geq 4$ if $f(x) + g(x) = a + b\sqrt{x + c} \qquad \frac{a + b}{c}$

$t = \sqrt{x - 4} \rightarrow f(x) = t + 2$

$\hookrightarrow g(x) = |t - 2| \rightarrow x \geq 4 \rightarrow t = \sqrt{x - 4} \geq 0 \rightarrow t - 2$

$f(x) + g(x) = (t + 2) + (t - 2) = 2t = 2\sqrt{x - 4}$

$\hookrightarrow c = -4 \qquad \frac{a + b}{c} = -\frac{2}{4} = -\frac{1}{2}$

$a + b = 2$

٩- $f(x)=\frac{x+٢}{x^٢-٤x+٣}$ ، $g(x)=\{(٢,١),(١,٠),(٣,٥),(٥,٤)\}$

$\frac{g}{f}=?$ ، $\frac{g(x)}{f(x)}=g(x)\times\frac{١}{f(x)} \rightarrow ١\times\frac{١}{-٤}=-\frac{١}{٤}$

$٤\times\frac{١}{\frac{٢}{٣}}=٦$

$g(٢)=١ \quad f(٢)=\frac{٤}{-١}=-٤$

$g(١)=٠ \quad f(١)=$ غ ق

$g(٣)=٥ \quad f(٣)$ غ ق

$g(٥)=٤ \quad f(٥)=\frac{٢}{٣}$

$\frac{g}{f}=\{(٢,-\frac{١}{٤}),(٥,٦)\}$

١٠-

$f(x)=\{(١,٢),(٣,-١),(٤,٢),(-١,٦)\}$

$g(x)=\{(-٢,٣),(١,-١),(٣,٢),(-١,٥)\}$

$f(٢x)=\{(٠.٥,٢),(١.٥,-١),(٢,٢),(-٠.٥,٦)\}$

$f(x^٢)=\{(\pm١,٢),(\pm\sqrt{٣},-١),(\pm٢,٢)\}$

$٢g^٢(x)+١=\{(-٢,١٩),(١,٣),(٣,٩),(-١,١)\}$

$\frac{٢f}{g}=\{(١,-٤),(٣,-١)\}$

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