

$$f(n) = \left(\frac{1}{n}\right)^{n-2} - 1 \rightarrow f(n+1) = \left(\frac{1}{n+1}\right)^{n+1-2} - 1 = \left(\frac{1}{n+1}\right)^{n-1} - 1$$

$$g = \sqrt{n^2 \cdot f(n+1)} = \sqrt{n^2 \left(\left(\frac{1}{n+1}\right)^{n-1} - 1\right)} \rightarrow n^2 \left(\left(\frac{1}{n+1}\right)^{n-1} - 1\right) \geq 0$$

$n=0 \leftarrow \quad \quad \quad \leftarrow \left(\frac{1}{n+1}\right)^{n-1} = 1 \rightarrow n-1=0 \Rightarrow n=1$

$$\frac{0}{-} \quad \frac{1}{+} \quad \frac{-}{-} \rightarrow D_f = [0, 1]$$

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$$f(n) = \sqrt{n+|n+1|} \rightarrow f(-n) = \sqrt{-n+|-n+1|} \rightarrow |2-n|-n \geq 0$$

$$\rightarrow 2-n \geq n \rightarrow 1 \geq n$$

$$\rightarrow 2-n \leq -n \rightarrow 2 \leq 0 \rightarrow \text{غیر ممکن} \Rightarrow D_f = (-\infty, 1]$$

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$$g = \sqrt{\frac{n-1}{f(n)}} \rightarrow \frac{n-1}{f(n)} \geq 0$$

$\rightarrow n = -2, 2, 4$

$$\frac{-2}{-} \quad \frac{1}{+} \quad \frac{2}{-} \quad \frac{4}{+} \quad \frac{-}{-}$$

$$D_f = (-2, 1] \cup (2, 4)$$

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$$g = \frac{1}{9n^2 - 4n^2 - 2n - 2} \rightarrow 9n^2 - 4n^2 - 2n - 2 \neq 0 \rightarrow 5n^2 - 4n - 2 \neq 0$$

مخرج ریشه نگیرد.

$$(5n^2 - 4n + 1) - (n^2 - 2n - 2) \neq 0$$

$$\frac{(5n^2 - 4n + 1)^2}{(n^2 - 1)^2} - \frac{(n^2 - 2n - 2)^2}{(n^2 - 1)^2}$$

$$(5n^2 - 4n + 1)^2 - (n^2 - 2n - 2)^2 \neq 0 \rightarrow 4n^2 - 1 = n + 2 \Rightarrow$$

$$4n^2 - n - 3 = 0 \rightarrow 4n^2 - n - 3 = 0$$

$$4n^2 - n - 3 = 4n^2 + an + b \Rightarrow a = -1, b = -3$$

ریشه دلتا را به هم برابر است.

$$\sqrt{-(a+b)} = \sqrt{-(-1-3)} = \sqrt{4} = 2$$

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$$g = \sqrt{f(n) - f\left(\frac{1}{n}\right)} = \sqrt{n^2 + n - \frac{4}{n^2} - \frac{1}{n}} \rightarrow n^2 + n - \frac{4}{n^2} - \frac{1}{n} \geq 0 \Rightarrow \frac{n^4 + n^5 - 4n^2 - 4}{n^2} \geq 0$$

$$n^4 + n^5 - 4n^2 - 4 \xrightarrow{n^2=t} t^2 + t^3 - 4t - 4 = t^3 - 4t + t^2 - 4t = (t-2)(t^2 + 2t + 4) + t(t-2)$$

$$\rightarrow (t-2)(t^2 + 2t + 4 + t) = (t-2)(t^2 + 3t + 4)$$

$t=2 \Rightarrow n^2=2 \leftarrow \quad \quad \quad \leftarrow \Delta = 9 - 4 = 5$

$n = \pm \sqrt{2}$

$$\frac{-2}{-} \quad \frac{0}{+} \quad \frac{2}{-}$$

$$D_f = [-\sqrt{2}, 0) \cup (2, +\infty)$$

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$$f(x^r + x) = rx^r - 1$$

$$\left. \begin{aligned} f(r) = f(x^r + x) \rightarrow x=1 &\leadsto r(1)^r - 1 = 1 \\ f(1) = f(x^r + x) \rightarrow x=r &\leadsto r(r)^r - 1 = r \end{aligned} \right\} f(r) = f(1) = 1 + r \in \Lambda$$

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$$f(n) = x^r - 4x^r + 14n - 1 + 1 = (n-r)^r + 1$$

$$g(n) = x^r + 4x^r + 14n + 14 - 14 = (n+r)^r - 14$$

$$\frac{g(\sqrt{r}-r)}{f(\sqrt{r}+r)} = \frac{(\sqrt{r}-r)^r - 14}{(\sqrt{r}+r)^r + 14} = \frac{r-14}{r+14} = \frac{-r}{14} \quad \left(-\frac{r}{14}\right)$$

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$$f(n) = \sqrt{(n-\varepsilon) + \varepsilon\sqrt{n-\varepsilon} + \varepsilon} \rightarrow f(n) = \sqrt{(\sqrt{n-\varepsilon} + r)^2} = |\sqrt{n-\varepsilon} + r|$$

$$g(n) = \sqrt{(n-\varepsilon) - \varepsilon\sqrt{n-\varepsilon} + \varepsilon} \rightarrow g(n) = |\sqrt{n-\varepsilon} - r|$$

$$f(n) + g(n) = a + b\sqrt{n+c} \rightarrow$$

$$\varepsilon \in \Lambda \rightarrow \sqrt{n-\varepsilon} + r - \sqrt{n-\varepsilon} + r = \varepsilon$$

$$n \notin \Lambda \rightarrow \sqrt{n-\varepsilon} + r + \sqrt{n-\varepsilon} - r = r\sqrt{n-\varepsilon} \rightarrow r\sqrt{n-\varepsilon}, a+b\sqrt{n+c} \Rightarrow a=0, b=r, c=-\varepsilon$$

$$\frac{a+b}{c} = \boxed{-\frac{1}{r}}$$

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$$g(n) = \{f(r, 1), (1, 1), (r, \omega), (1, \varepsilon)\}$$

$$f(n) = \frac{x+r}{x^r - \varepsilon n + r} \rightarrow f(r) = \frac{r+r}{r - 1 + r} = \frac{\varepsilon}{-1} = -\varepsilon \quad f(1) = \frac{1+r}{1 - \varepsilon + r} = \frac{r}{\varepsilon} \in \mathbb{Q}$$

$$f(r) = \frac{r+r}{r - 1 + r} = \frac{\omega}{0} \in \mathbb{Q} \quad f(1) = \frac{1+r}{1 - \varepsilon + r} = \frac{r}{\varepsilon}$$

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$$\frac{g}{f} = \left\{f(r, -\frac{1}{\varepsilon}), (1, \frac{1}{\varepsilon}), (r, \frac{\omega}{\varepsilon}), (1, \frac{\varepsilon}{r})\right\}, \boxed{\left\{f(r, -\frac{1}{\varepsilon}), (1, \frac{1}{\varepsilon})\right\}}$$

$$a) f(n) = \left\{f\left(\frac{1}{r}, r\right), \left(\frac{r}{r}, -1\right), (r, r), \left(-\frac{1}{r}, r\right)\right\}$$

$$b) f(n) = \left\{(1, r), (\sqrt{r}, -1), (r, r)\right\}$$

$$c) rg^r(n+1) = \{(1-r, r(n+1)), (1, r(n+1)), (r, r(n+1)), (-1, r(n+1))\} = \{(r, n), (1, r), (r, n), (-1, 1)\}$$

$$d) \frac{rf}{g} = \left\{(1, \frac{\varepsilon}{-1}), \left(-1, \frac{1}{\varepsilon}\right), \left(r, \frac{r}{\varepsilon}\right)\right\} = \left\{(1, -\varepsilon), (r, -1)\right\}$$

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