شماره تکلیف: ۵ — یازدهم دختر A — دیانا نامی

1)

$f(x) = (\frac{1}{2})^{x-2} - 1 \qquad y = \sqrt{x^3 f(x+1)} = \sqrt{x^3 \times [(\frac{1}{2})^{x-1} - 1]} \geq 0$

$\begin{cases} x = 0 \\ (\frac{1}{2})^{x-1} - 1 = 0 \to (\frac{1}{2})^{x-1} = 0 \to x = 1 \end{cases}$

$\begin{array}{ccccc} & 0 & & 1 & \\ - & \phi & + & \phi & - \end{array} \qquad D = [0, 1]$

---

2)

$f(x) = \sqrt{x + |x+2|} \qquad f(-x) = \sqrt{-x + |-x+2|} \geq 0 \to$

$-x + 2 = 0 \to x = 2$

$\begin{array}{ccc} & 2 & \\ + & \phi & - \end{array}$

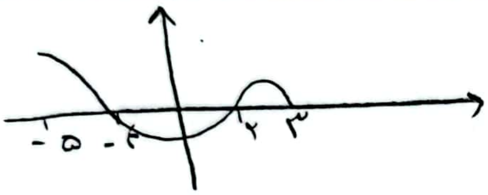
1) $x \leq 2 \to -x + (-x+2) = -2x + 2 \geq 0 \to 2x \leq 2 \to x \leq 1 \to (-\infty, 1]$

2) $x \geq 2 \to -x + x - 2 = -2$ ✗

$\to D = (-\infty, 1]$

---

3)



$y = \sqrt{\frac{x-1}{f(x)}} \geq 0$

$\begin{cases} x - 1 \geq 0 \to x \geq 1 \\ f(x) > 0 \to (-\infty, -4) \cup (2, 3) \end{cases} \to (2, 3)$

$\begin{cases} x - 1 \leq 0 \to x \leq 1 \\ f(x) < 0 \to (-4, +2) \end{cases} \to (-4, 1)$

$\to D_y = (2, 3) \cup (-4, 1)$

---

4)

$y_1 = \frac{1}{9x^4 - 7x^2 - 4x - 3}$

---

5)

$f(x) = x^3 + x \qquad y = \sqrt{f(x) - f(\frac{4}{x})}$

$f(x) - f(\frac{4}{x}) = x^3 + x - ((\frac{4}{x})^3 + \frac{4}{x}) = x^3 + x - \frac{64}{x^3} - \frac{4}{x} = \frac{x^6 + x^4 - 64 - 4x^2}{x^3} =$

$= (x^2 - 4)(\underbrace{x^4 + 5x^2 + 16}_{\Delta < 0})$

$\begin{array}{ccccc} & -2 & & 2 & \\ + & \phi & - & \phi & + \end{array}$

1) $|x| > 2 \to (+) \longrightarrow x > 0 \to x \geq 2$

2) $|x| < 2 \to (-) \longrightarrow x < 0 \longrightarrow -2 \leq x < 0$

3) $|x| = 0 \to 0 \longrightarrow x \neq 0$

$D = [-2, 0) \cup [2, +\infty)$

$f(x^3+x)=2x^2-1$

$f(2)+f(10) \xrightarrow[**]{*} 2+1=3$

$x^3+x=10 \longrightarrow x=2$ (*)

$x^3+x=2 \longrightarrow x=1$ (**)

$f(x)=x^3-6x^2+12x$

$g(x)=x^3+9x^2+27x$

$$\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)} \overset{(1)}{\underset{(2)}{=}} \frac{t^3-27}{t^3+8}=\frac{7-27}{2+8}=\frac{-20}{10}=-2$$

(1) $\sqrt[3]{7}=t \Rightarrow g(t-3)=(t-3)^3-6(t-3)^2+12(t-3)=$

$(t-3)\left((t-3)^2-6(t-3)+12\right)=(t-3)(t^2-6t+9+9t\cancel{-27}\cancel{+27})=$

$(t-3)(t^2-3t+9)=t^3-27$

(2) $\sqrt[3]{2}=t \rightarrow f(t+2)=(t+2)^3-6(t+2)^2+12(t+2)=$

$(t+2)\left((t+2)^2-6(t+2)+12\right)=(t+2)(t^2-2t+4)=t^3+8$

$f(x)=\sqrt{x+4\sqrt{x-4}}$ $\qquad g(x)=\sqrt{x-4\sqrt{x-4}}$

$f(x)=\dfrac{x+2}{x^2-4x+3}$

$x^2-4x+3=0 \rightarrow (x-1)(x-3)=0 \longrightarrow x\neq 1,\ x\neq 3$

$D_g=\{2,\cancel{1},\cancel{3},0\} \longrightarrow g(2)=1,\ g(0)=4 \qquad f(2)=\frac{4}{-1}=-4,\ f(0)=2/3$

$\frac{g}{f}=\{(2,-\frac{1}{4}),(0,6)\}$

10)

$$f(2x) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (2, 2), \left(-\frac{1}{2}, 4\right) \right\}$$

$$f(x^2) = \{ (1, 2), (9, -1), (16, 2), (1, 4) \}$$

$$2g^2(x) + 1 = \{ (-2, 19), (1, 4), (4, 9), (-1, 1) \}$$

$$\frac{2f}{g} = \{ (1, -4), (4, -1) \}$$