

نام و نام خانوادگی فایده سنجی پاسخنامه تشریحی تکلیف شماره ۱۰۰ کلاس یا نرجه خنجره A

$$x^3 f(x+1) \geq 0 \rightarrow f(x+1) = \left(\frac{1}{x}\right)^{x-1} - 1 \Rightarrow x^3 \left(\left(\frac{1}{x}\right)^{x-1} - 1\right) \geq 0$$

$\begin{array}{c} 0 \quad 1 \\ - \quad + \quad + \quad - \end{array}$

$\boxed{[0, 1] = \text{دامنه}}$

$$f(-x) = \sqrt{-x+12-x} \Rightarrow -x+|12-x| \geq 0 \Rightarrow |12-x| \geq x$$

$\begin{array}{l} \rightarrow 12-x \geq x \Rightarrow \boxed{1 \geq x} \text{ (1)} \\ \rightarrow 12-x \leq -x \Rightarrow x \leq -12 \end{array}$

$D_{f(-x)} = (-\infty, 1]$

$$\frac{x-1}{f(x)} \geq 0$$

$\begin{array}{c} -\varepsilon \quad 1 \quad 2 \quad 3 \\ - \quad + \quad - \quad + \quad - \end{array}$

$D_y = (-\varepsilon, 1] \cup (2, 3)$

چون دامنه دو تابع برابرند یعنی باید شامل ۲ باشد. درواقع ۱ باشد

$$(3x^2 + ax + b)(3x^2 + cx + d) \rightarrow 9x^4 + 3cx^3 + 3dx^2 + 3ax^2 + acx^2 + adx + 3bx^2 + cbx + db$$

$\Rightarrow 9x^4 + 3cx^3 + 3dx^2 + 3ax^2 + acx^2 + adx + 3bx^2 + cbx + db$

$\begin{array}{l} db = -3 \\ c = -a \\ a(d-b) = 4 \\ 3d + a^2 + 3b = -7 \end{array} \Rightarrow \begin{array}{l} a = -1 \\ b = -3 \\ d = 1 \end{array}$

$\sqrt{-(-3-1)} = \sqrt{4} = 2$

$$f(x) = x^3 + x, f\left(\frac{\varepsilon}{x}\right) = \frac{4\varepsilon}{x^3} + \frac{\varepsilon}{x}$$

$f(x) - f\left(\frac{\varepsilon}{x}\right) \geq 0 \Rightarrow x^3 + x - \frac{4\varepsilon}{x^3} - \frac{\varepsilon}{x} \geq 0$

$x^3 + x = \frac{4\varepsilon}{x^3} + \frac{\varepsilon}{x} \xrightarrow{x^2 = t} t^3 + t = \frac{4\varepsilon}{t} + \frac{\varepsilon}{t} \Rightarrow (t-\varepsilon)(t^2 + \varepsilon t + 14) + t(t-\varepsilon) = 0$

$\Rightarrow (t-\varepsilon)(t^2 + \varepsilon t + 14) = 0 \Rightarrow x = \pm 2$

$\begin{array}{c} -2 \quad 2 \\ + \quad - \quad + \end{array} \quad D_y = (-\infty, -2] \cup [2, +\infty)$

$$x^3 + x = 2 \Rightarrow x = 1 \Rightarrow f(1) = 1(1)^3 - 1 = 0$$

$$x^3 + x = 10 \Rightarrow x = 2 \Rightarrow f(2) = 2(2)^3 - 1 = 7$$

$$\left\{ \begin{array}{l} f(1) + f(10) = \boxed{7} \end{array} \right.$$

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$$f(x) = x^3 - 4x^2 + 12x - 1 + 1 = (x-2)^3 + 1$$

$$g(x) = x^3 + 9x^2 + 27x + 27 - 27 = (x+3)^3 - 27$$

$$\frac{g(\sqrt[3]{27} - 3)}{f(\sqrt[3]{27} + 3)} = \frac{(\sqrt[3]{27} - 3 + 3)^3 - 27}{(\sqrt[3]{27} + 3 - 3)^3 + 1} = \frac{27 - 27}{2 + 1} = \frac{-27}{10} = \boxed{-\frac{27}{10}}$$

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$$\sqrt{x-2} = t \Rightarrow t^2 + 2 = x$$

$$f(x) = \sqrt{t^2 + 2t + 2} = t + 1$$

$$g(x) = \sqrt{t^2 - 2t + 2} = |t - 1|$$

$$f(x) + g(x) \xrightarrow{\substack{\text{با تغییر متغیر} \\ \boxed{2t} \checkmark}} \xrightarrow{\substack{\text{با تغییر متغیر} \\ \boxed{2t} \checkmark}} \Rightarrow 2t = \underbrace{(1)}_{a+b} \underbrace{\sqrt{x-2}}_c \Rightarrow \frac{a+b}{c} = \boxed{-\frac{1}{2}}$$

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$$D_f = \mathbb{R} - \{1, 3\} \text{ و } D_g = [0, 2] \xrightarrow[\substack{\text{با تغییر متغیر} \\ x=2, x=0}}{\substack{\text{با تغییر متغیر} \\ x=2, x=0}} \{0, \frac{2}{3}, (2, -2)\}$$

$$\xrightarrow[\substack{\text{با تغییر متغیر} \\ x=2, x=0}}{\substack{\text{با تغییر متغیر} \\ x=2, x=0}} \{(0, 2), (2, 1)\}$$

$$\frac{g}{f} = \{(0, 2), (2, -\frac{1}{2})\}$$

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$$\text{الف) } f(x) = \{(1, 2), (\sqrt{3}, -1), (2, 2), (-\frac{1}{2}, 4)\}$$

$$\text{ب) } f(x^2) = \{(1, 2), (\sqrt{3}, -1), (2, 2), 4\}$$

$$\text{ج) } 2g^2(x) + 1 = \{(1, 9), (3, 1), (2, 9), (-1, 1)\}$$

$$\text{د) } \frac{f}{g} = \{(1, -2), (3, -1)\}$$

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