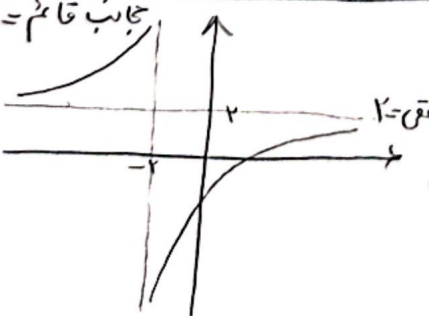
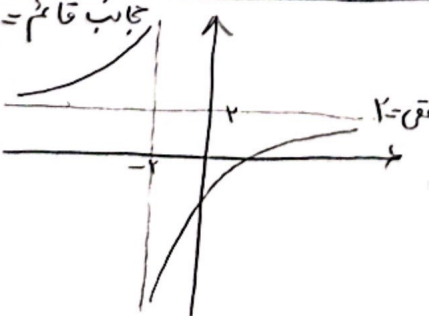
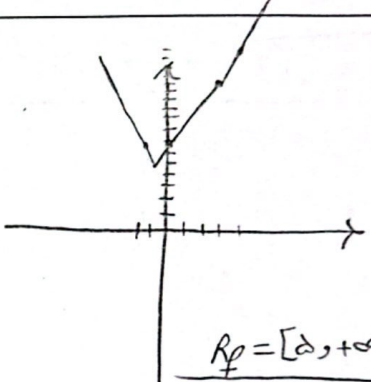
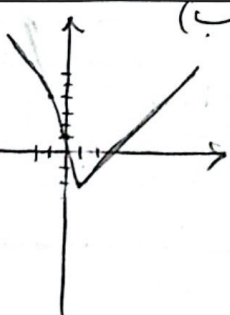


الف) $y = (n-1)^3 + 1 \rightarrow y-1 = (n-1)^3 \rightarrow \sqrt[3]{y-1} = n-1 \rightarrow n = \sqrt[3]{y-1} + 1 \rightarrow R_f = \mathbb{R}$ ب) $y = \frac{1}{n^2 - 2n} \rightarrow y(n^2 - 2n) = 1 \rightarrow y(n^2 - 2n) - 1 = 0 \rightarrow \Delta = (-2y)^2 - 4(y)(-1) = 4y^2 + 4y$ $\frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y^2 + 4y > 0 \rightarrow 4y(y+1) > 0 \rightarrow y < -1 \text{ or } y > 0 \rightarrow R_f = (-\infty, -1) \cup (0, +\infty)$	۱
الف) $y = n^2 - 4n + 10 \xrightarrow{\alpha > 0} n_{\min} = \frac{-b}{2a} = \frac{-(-4)}{2} = 2 \xrightarrow{\text{if } n=2} y = 6 \rightarrow R_f = [6, +\infty)$ ب) $y = -n^2 + 4n + 1 \xrightarrow{\alpha < 0} n_{\max} = \frac{-b}{-2a} = \frac{-4}{-2} = 2 \xrightarrow{\text{if } n=2} y = 9 \rightarrow R_f = (-\infty, 9]$ ج) $y = \sqrt{n^2 - 4n - 3} \xrightarrow{\alpha > 0} n_{\min} = \frac{-(-4)}{2} = 2 \xrightarrow{\text{if } n=2} y_{\min} = -1 \rightarrow R_f = [-1, +\infty) = [0, +\infty)$ د) $y = \sqrt{4n - n^2} \xrightarrow{\alpha < 0} n_{\max} = \frac{-b}{-2a} = \frac{-4}{-2} = 2 \xrightarrow{\text{if } n=2} y_{\max} = 2 \rightarrow R_f = [0, 2]$	۲
الف) تابع منگه مندر است $R_f = \mathbb{R}$ ب) $R_f = \mathbb{R}$ ج) $R_f = \sqrt{\mathbb{R}} = [0, +\infty)$ د) تابع توان دارد $R_f = [0, +\infty)$	۳
الف) $y = \frac{2n+1}{n-2} \rightarrow R_f = \mathbb{R} - \left\{\frac{1}{2}\right\} \Rightarrow R_f = \mathbb{R} - \left\{\frac{1}{2}\right\}$ ب) $y = \frac{2n+1}{n+1} \rightarrow R_f = \mathbb{R} - \left\{\frac{1}{2}\right\}$	۴
الف) $y = \sqrt{\frac{2n+2}{n+1}} \rightarrow R_f = \sqrt{\mathbb{R} - \left\{\frac{1}{2}\right\}} = [0, +\infty) - \left\{\sqrt{\frac{1}{2}}\right\}$ ب) $y = \sqrt{\frac{2n+1}{n+1}} \rightarrow R_f = \sqrt{\mathbb{R} - \left\{\frac{1}{2}\right\}} = [0, +\infty)$	۵
الف)  ب) 	۶

$R_f = (-\infty, -r] \cup [r, +\infty)$	(الف)	
$y = \frac{n^q + 1}{n^p} \rightarrow y = n^q + \frac{1}{n^p} \Rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$	(ب)	✓
$y = \frac{\sqrt[n]{n^r} + 1}{\sqrt[n]{n}} \rightarrow y = \sqrt[n]{n} + \frac{1}{\sqrt[n]{n}} \Rightarrow R_f = (-\infty, -r] \cup [r, +\infty)$	(ج)	
$y = \sqrt{n} + \frac{1}{\sqrt{n}} \Rightarrow R_f = [r, +\infty)$	(د)	
$y = n^r + \frac{1}{n^r + r} \xrightarrow{r+1-p} n^{r+1} + \frac{1}{n^{r+1} + r} - r \rightarrow R_f = [\frac{1}{r+1}, +\infty)$ $r + \frac{1}{r} - r = \frac{1}{r} \Rightarrow \text{min}$	(هـ)	
$y = \frac{n^r + d}{\sqrt{n^r + r}} \rightarrow y = \frac{n^r + r + 1}{\sqrt{n^r + r}} \rightarrow y = \frac{n^r + r}{\sqrt{n^r + r}} + \frac{1}{\sqrt{n^r + r}} \rightarrow y = \sqrt{n^r + r} + \frac{1}{\sqrt{n^r + r}}$ $y_{\min} = \sqrt{r} + \frac{1}{\sqrt{r}} \rightarrow R_f = [\frac{d}{r}, +\infty)$	(و)	1
$y = n-r + n+r $ $f(n) = \begin{cases} n-r+n+r & n \geq r \\ -n+r+n+r & -\frac{r}{2} \leq n \leq r \\ -n+r-n-r & n \leq -\frac{r}{2} \end{cases} \rightarrow f(n) = \begin{cases} 2n & n \geq r \\ r & -\frac{r}{2} \leq n \leq r \\ -2n & n \leq -\frac{r}{2} \end{cases}$	 $R_f = [d, +\infty)$	9
$y = n-r - n+1 $ $f(n) = \begin{cases} n-r & n \geq 1 \\ -n+1 & -1 \leq n \leq 1 \\ -n+r & n \leq -1 \end{cases}$	 $R_f = [-r, +\infty)$	
	$y = n-r + n+r $ (دو) $f(n) = \begin{cases} n & n \geq r \\ n+r & -r \leq n \leq r \\ -n & n \leq -r \end{cases}$	10